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ABSTRACT

Reported are the results of a study whose aim was to compare the standards for university entrance in the various member states of the Council of Europe. Presented is an analysis and comparison of the academic secondary mathematics education curriculum at the most specialized level, focusing on what is required to be known to pass successfully the "leaving examination." Following an introductory chapter is a commentary on the aims of mathematics teaching, with specific quotations from official publications in the various countries. An application of Bloom's taxonomy is discussed in terms of mathematics objectives and examinations; the "leaving examination" is discussed separately. Comparisons of syllabi and a summary of proposed changes in the syllabi of various countries are presented, followed by some comments on teaching method and data on teachers. Twelve appendices include a concise analysis of mathematics topics common to and those specific to the member countries, analyses of time allocations, and other specific data reporting survey results discussed in each chapter. (MS)

Council of Europe

COUNCIL
FOR
CULTURAL
CO-OPERATION

EUROPEAN CURRICULUM STUDIES

No 1 : MATHEMATICS

1968

The Council of Europe was established by ten nations on 5th May 1949, since when its membership has progressively increased to eighteen. Its aim is "to achieve a greater unity between its Members for the purpose of safeguarding and realising the ideals and principles which are their common heritage and facilitating their economic and social progress". This aim is pursued by discussion of questions of common concern and by agreements and common action in economic, social, cultural, scientific, legal and administrative matters.

The Council for Cultural Co-operation was set up by the Committee of Ministers of the Council of Europe on 1st January 1962 to draw up proposals for the cultural policy of the Council of Europe, to co-ordinate and give effect to the overall cultural programme of the organisation and to allocate the resources of the Cultural Fund. It is assisted by three permanent committees of senior officials: for higher education and research, for general and technical education and for out-of-school education. All the member Governments of the Council of Europe, together with Spain and the Holy See which have acceded to the European Cultural Convention, are represented on these bodies[1].

In educational matters, the aim of the Council for Cultural Co-operation (CCC) is to help to create conditions in which the right educational opportunities are available to young Europeans whatever their background or level of academic accomplishment, and to facilitate their adjustment to changing political and social conditions. This entails in particular a greater rationalisation of the complex educational process. Attention is paid to all influences bearing on the acquisition of knowledge, from home television to advanced research; from the organisation of youth centres to the improvement of teacher training. The countries concerned will thereby be able to benefit from the experience of their neighbours in the planning and reform of structures, curricula and methods in all branches of education.

Since 1963 the CCC has been publishing, in English and French, a series of works of general interest entitled "Education in Europe", which record the results of expert studies and intergovernmental investigations conducted within the framework of its programme. A list of these publications will be found at the end of this volume.

These works are now being supplemented by a series of "companion volumes" of a more specialised nature, including catalogues, handbooks, bibliographies etc., as well as selected reports of meetings and studies on more technical subjects. These publications, to which the present study belongs, are listed at the end of the volume.

General Editor: The Director of Education and of Cultural and Scientific Affairs, Council of Europe.
Strasbourg - France.

1. For complete list, see back of cover.

EUROPEAN

CURRICULUM STUDIES

No 1: MATHEMATICS

EUROPEAN CURRICULUM STUDIES

(In the Academic Secondary School)

by
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Oxford University Department of Education

Council for Cultural Co-operation
Strasbourg
1968

" A mathematical training is a possession to which every human being has a right, no matter what their race, sex, situation in life, or occupation."

Brochure No. 133. The National Paedagogical Institute, Paris, 1957, Page 18.

"Everywhere we come across mathematics : the paper we write on is standardised and we bet on football matches, we grind our coffee and in our cars we speed around the bends in a motorway, we build vertiginous bridges and we take out life insurances, we leave our money to bear interest in the savings bank and we lose it gambling, we grouse about statistics and we can't live without them..."

Menniger, *Mathematik in Deiner Welt*. Göttingen 1958.

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INTRODUCTION

This small book is one of the outcomes of the Oxford-Council of Europe Evaluation Study, undertaken by Oxford University Department of Education on behalf of the Committee for General and Technical Education of the Council for Cultural Cooperation, with the generous backing of that organisation and of the Gulbenkian Foundation. The aim of the study is to compare the standards required for university entrance in the various member states of the Council of Europe : this has necessitated not only an investigation of the academic secondary leaving examination in those countries. but detailed comparisons of the curriculum. As the work has proceeded it has become clear that the study of the curriculum is of paramount interest to all countries. Countries, whether themselves highly industrialised or still mainly agricultural, can learn much from each other as to examining techniques, curriculum content and teaching method.

It is hoped that this first volume will be one of a series entitled European Curriculum Studies. If it is successful, some kind of curriculum bulletin, listing major changes each year, would be a convenient means of keeping it up to date.

This book deals solely with mathematics in the academic secondary school or stream, and then only with the curriculum for those taking the subject at the most specialised level, e.g. Section C of the *classe terminale* of the French *lycée*, the *Oberstufe* of the *Mathematischen-Wissenschaftlichen Gymnasien* etc. The curriculum analysed is not necessarily confined to those topics studied in the last year, or last two or three years of schooling, but represents the best judgement of subject experts as to what is actually required to be known in order successfully to pass the academic secondary leaving examination. Although every care has been taken to verify data given, both by consultation with experts and by reference to the official publications that were available to us, it is inevitable that some errors have crept in. We apologise in advance for these. To have submitted the draft of the MS for approval to every member government, at a time when curricula and syllabuses are rapidly evolving, would have been timewasting. It is hoped, instead, that member governments will send in amendments for correction in any subsequent edition.

I. THE ACADEMIC SECONDARY CURRICULUM AND MATHEMATICS

Whilst educationists have long since abandoned an "encyclopedic" concept of the curriculum in which, as in medieval times, pupils were acquainted with the main facets of every aspect of knowledge, the belief still largely persists that it is possible to instruct the academic secondary pupil in what it is considered "essential" for him to know. Unfortunately, what is "essential" is capable of various definitions. Is what has to be learned necessary for future higher education, for personal development or for later employment? Most educationists would agree that in drawing up a curriculum their intentions are manifold ; they would likewise agree that the finished product is more often than not the result of a compromise. The German curriculum theory of *exemplarisches Lernen* is one such compromise : it recognises that today "every subject can have only paradigmatic significance". The corpus of knowledge is so vast, the sum of information and skills required in post-secondary education or employment is changing so rapidly that one must be content if the pupil has been "initiated" into the four main realms of experience : philosophical-religious, historical-social, scientific-technological and aesthetic. By exemplifying some aspect of knowledge, mode of thinking or skill, the pupil will be brought to appreciate the variety and complexity of a discipline. By contrast, the English viewpoint is more rigorous : the theory of "specialisation" assumes that "study in depth" of very few subjects will equip the pupil later to undertake study in other disciplines. This is also diametrically opposed to the "survey" theory of the curriculum, which has had its vogue in America, whereby a broad conspectus of knowledge was reviewed, but necessarily only superficially. It is a measure of the uncertainty in curriculum matters today that countries find it difficult to pin their faith to any one theory of the curriculum, whether "encyclopedic", "essential", "paradigmatic", "specialised" or demanding a broad "survey" of knowledge.

The present flux in mathematics teaching is characteristic of that uncertainty. It might be argued that the very nature of the subject is changing in some respects. What is certain is that its importance remains undiminished. It is one of the few subjects studied in the academic secondary school by all pupils for at least the first four or five years of the course, and in many countries right up to the time of leaving school. It is also one of the few subjects for which a qualification, either at a higher or a lower level of study, is required in practically every country, before a pupil can proceed to higher education. Its central place in the curriculum is therefore apparent.

Why it should be considered as an indispensable element in general culture is likewise plain: it is the symbolic language of science, one of the 'communicators', mastery of which is essential for the acquisition of further knowledge in many other fields.

Despite its assured place in the curriculum, it is nevertheless one of the subjects about which much rethinking is going on. Not only is "new mathematics" being introduced practically everywhere and at all levels of education, but a new conception of the unity of the subject has entailed much syllabus revision and refurbishing of teaching method. Artificial division of the subject into elements such as "algebra", "geometry", "calculus" etc. militates against the logical unity of the subject. Instead, pupils are now often presented with problems which, for example, may only be resolved by using algebra, arithmetic and trigonometry at one and the same time.

Not only are teachers beginning to insist upon this *internal* unity of mathematics, but they are also beginning to stress its unity with other subjects, particularly with the science disciplines. The practicality of the notion of vectors for students of physics, for example, does not need to be stressed. Indeed, there is a carry-over into the realm of the social sciences: the psychologist, the economist, the sociologist cannot function efficiently without being able to manipulate statistics or consider their findings in the light of probability theory. Even in the field of the humanities mathematics is increasingly used: from linguistics to archeology, from textual criticism to Biblical exegesis, mathematical techniques are being employed. In more recent advances in knowledge, in subjects such as information techniques and data retrieval, mathematics is indispensable. Indeed, it can be argued that an inevitable concomitant of the 'knowledge explosion' must be a felt need to order, arrange and quantify — procedures that are impossible without the use of mathematical techniques.

II. THE AIMS OF MATHEMATICS TEACHING

With the notable exception of the United Kingdom, all countries lay down in official publications the aims, directives and programmes for mathematics at the academic secondary level. This information, however, varies very much according to the degree of detail that is to be found. England, which works through eight examining boards, gives the syllabuses in considerable detail, but little, if any, directives as to how this syllabus is to be taught [1]. On the other hand, Germany (Nordrhein-Westfalen) goes into considerable detail both as to aims and directives *(Richtlinien)*.

A selection from the more important publications which set out these aims, directives and programmes is given in Appendix I (page 47).

A study of the aims of mathematics teaching in a number of countries [1] as set out in these official syllabuses and directives reveals a wide diversity [2]. There is no doubt, however, that most countries would subscribe to all the aims mentioned by all other countries. What is interesting is the emphasis put on them in each country. It should be noted that these aims are couched in very general terms and, as will be seen later, are not necessarily reflected in the specific aims which most countries subscribe to when it comes to the examination of mathematics.

The first group of aims may be categorised as dealing with mathematics itself. The majority of countries subscribes to the view that pupils should be made aware of the significance of mathematics and of the characteristics of the subject. They are also concerned with initiating them into the modes of mathematical thinking, which some distinguish from the objective of teaching mathematics as an intellectual discipline. A broad definition of mathematical thinking might be the ability to think quantitatively as well as qualitatively.

Mathematics should also be taught, all countries agree, from the *instrumentalist* viewpoint. By this is meant not only the use of mathematics in the day-to-day business of life, but also the use of the subject in other academic fields of knowledge, particularly science, technology, sociology and economics. Teaching should be orientated not only to show the relevance of mathematics to these natural and social sciences but also to show the coherence that exists between it and related fields.

Despite the many strictures levelled by psychologists against the theory of transfer of training, it would seem most countries in fact believe that mathematics acts as an intellectual discipline, imparting a mental training to those pupils who study it. Among the qualities which are alleged to be sharpened by mathematics are pre-eminently logic (mentioned by no less than five countries), imagination and creativity (mentioned by three countries), and qualities of precision, clarity, resourcefulness and judgment. This belief in mental training is linked in some countries with the enrichment of the general culture of the pupil. In France, for example, the general instructions of 1st October 1946 (p. 5) [1] [3] sum up this viewpoint as follows :

" To aim at educating the mind and at imparting a general culture : to promote the untrammelled and complete development of the child's faculties, fostering in him the growth of all that constitutes the excellence of mankind; intelligence, affection, character, moral sense, an inclination towards righteousness. "

1. With regard to the United Kingdom, the diversity of examing boards in England, and the fact that the Scottish and Northern Ireland education systems differ in fundamental aspects has created difficulties in this study. In all references to the United Kingdom, except where otherwise stated, information is not based specifically on any of the constituent countries, nor on any particular examining authority. Instead, the best judgment of British experts has been drawn upon for matters such as aims and syllabus content, etc

2. In Austria, Belgium, Denmark, France, Federal Republic of Germany (Nordrhein-Westfalen), Luxembourg, Sweden, Netherlands, Italy and Spain.

3. Numbers in square brackets refer to documents mentioned in the list of sources given on page 49.

Particularly interesting to the English observer is the assertion that mathematics not only heightens *intellectual* but also *moral* perception and is thus a part of the development of a pupil's character. He is placed in a relationship which demands from him nothing less than perfection, which is set up as an ideal. The French perhaps express this moral aspect best when they use phrases as 'to teach one to distinguish the true from the false, amid the contradictions of mankind'; 'to examine all things, relating them to their principles' and 'to reason about taking into account only those facts clearly and appropriately confirmed'; 'to observe, measure, to criticise one's own observations by proceeding to a rigorous verification, to complete enumerations and to make decisive experiments'; 'to treat a complex question in its totality and to analyse it in its details by putting each element into its place and into its proper plane'; 'to expound clearly, methodically and objectively any matter'; 'to read a document and to read between the lines of it, grasping exactly its significance, its application and its value'; 'to express both or. , and in writing all that one has to say, developing one's thoughts to their logical limits but without drawing false conclusions'. Thus to the intellectual exactness of a Descartes is added the moral sensibility of a Pascal.

To the intellectual and moral aspects of mathematics teaching, the Germans and Italians assert its value in increasing the *aesthetic* appreciation of the pupil. The German instructions (Nordrhein-Westfalen) express this by saying that the task of the mathematics teacher is 'to give insight into the many beautiful relationships in numbers and figures that exist and to recognise relationships of size and form also in the outside world'.

To these previous dimensions of mathematical values the Germans also add a *philosophical* one. They wish the pupil to become aware of the vastness, and yet the limitations, of mathematical thinking. This is envisaged as a two-stage operation, the first of which is to make the pupil aware of the logicality of the foundations and methods used in the subject. He is no longer to accept the premises that were put before him at an earlier stage but is to realise on what they are based. This first stage is to be followed by the philosophical stage p)per in which theoretical considerations of epistemology and ontology are to be brought to bear. The official directives say, however, that the nature of the subject and the way of teaching it precludes anything but an occasional allusion to this second stage, which must be reserved for the subject discipline of philosophy proper. It must be remembered that in the Federal Republic of Germany philosophy is an optional choice in the *Abitur*. In school systems where no philosophy is systematically studied, such as in the United Kingdom, it is doubtful whether this philosophical aspect of mathematics can in fact play much part. The Germans, however, say that mathematical study at this level can lead on to consideration of the different conceptions of the nature of the subject (for example, formalism and intuition). The pupil can be made to realise that it rests neither upon pure thought (logic) nor on mere experience (empiricism). He must also be confronted with the question whether mathematics is like any other branch of knowledge in that it is bound to certain subjects and conditions.

Lastly, all countries stress the value of mathematics as a *means of developing the powers of expression*. Mathematics is, after all, a symbolic language which should be capable of being expressed in words. The Belgian Catholic directives for the teaching of mathematics, for example, assert categorically: 'Mathematics teachers have a built-in advantage: the subject that they teach requires in a very special way a precise language. They must therefore show themselves meticulous not only for the exactness of the subject matter but for the correctness, and even the elegance of the form'. The Belgian official instructions [4] p. 6. are couched in similar terms. 'The teacher', they say, 'will therefore awaken (in his pupils) concern for exactness by demanding the exact word, correct definitions and correct statements, sober, clear and complete replies'. Likewise the Germans are not so much concerned with the accuracy of the mathematical calculations, but are anxious that the pupils should be able to express a logical sequence of thought in a correct and convincing way. The English relate mathematical fluency to general fluency

TABLE I

Aims of mathematics teaching

	Au	BgC	BgS	DK	Fr	FRG	Ir	It	Lu	Ne	Sd	Sp	UK
Mathematics for use..	x	x	x	x	x	x					x		x
Significance and characteristics of mathematics	x			x	x	x			x				
Coherence with other subjects e.g. science, technology, economics	x	x	x	x	x	x	x	x	x	x	x	x	x
Mental training [1]	x	x	x	x	x	x		x	x	x	x		x
Modes of mathematical thinking	x	x	x	x	x	x		x			x	x	x
Culture générale			x		x	x		x		x			
Character and moral training			x										
Developing powers of expression	x	x	x		x	x			x		x	x	x
Inculcation of aesthetic sense						x		x					
Mathematics as an approach to philosophical thinking ..						x							

1. " clarity " — Denmark, France
" resourcefulness " — Denmark
" imagination " — Denmark, Italy, Belgium C
" precision " — France, Belgium C
" logic " — Denmark, France, Italy, Belgium C, Sweden, Luxembourg, United Kingdom
" judgment " — Belgium S

Key

Au = Austria
BgC = Belgium (Catholic)
BgS = Belgium (official)
Dk = Denmark
Fr = France
FRG = Federal Republic of Germany
Ir = Ireland
It = Italy
Lu = Luxembourg
Ne = Netherlands
Sd = Sweden
Sp = Spain
U.K. = United Kingdom

of expression : ' Mathematics, itself a language, wilts into a dull and often incomprehensible confusion if it is divorced from fluent expression in the ordinary language of speech ' [13] p. 124. It is obvious that both oral and written work in mathematics can provide excellent practice in clear expression.

What emerges from this brief analysis is the close similarity of the aims of mathematical teaching in the various countries of the Council of Europe. There is broad agreement as to what might be termed the ' narrow ' aims of mathematics and then insistence in degrees, varying according to the country, on those aspects of mathematics which can promote the intellectual, liberal, aesthetic and philosophical development of the pupils. What is of interest to the dispassionate observer is not the nature of the aims, which are surely incontrovertible, but the linkage that should exist between these aims and the syllabus content as such. It would seem that the attainment of general objectives is almost more a question of method — the way in which the content of the syllabus is taught — rather than the syllabus itself.

The following table shows in summary form the aims of mathematics teaching in various countries. The fact that a country may not specifically mention a particular aim is not of course to imply that this aim is not one of its objectives. The purpose of the table is merely to show, first the common ground of agreement that patently exists as to aims, and secondly, the different emphases that are also apparent.

The following quotations regarding aims are representative extracts from official publications in various countries.

(1) BELGIUM

Aims

A

Mathematics has great utilitarian value if we consider the contribution it makes to the study and development of other sciences.

Its fundamental aim is to give the pupil the ability to deal with problems not only in higher education, but also in everyday life. We should therefore awaken in the pupil a critical spirit and develop his qualities of analysis and synthesis.

A good mathematical education should rely more on judgment rather than on memory.

(Summarised from : Ministère de l'Instruction Publique — *Mathématiques*, 1955, [4] pp. 5-6)

Each new theory introduced should be illustrated by applications. The ultimate aim of the mathematics course is the development of the powers of logic and the capacity for analysis and observation, and the exercise of creative imagination.

The teacher should seize every opportunity to promote the development of a mathematical spirit and to enrich the general culture of the pupil. He should indicate the liaisons between the various branches of mathematics and the broad outlines of the history of the science of numbers.

One cannot insist too much on the need for cultivating in the pupil care for correct and precise language, constant preoccupation with exactitude of expression " which alone is adequate ", in the words of La Bruyère.

The rigour demanded in mathematics contributes to the study of other disciplines.

(*ibid.*, p. 24 — summarised)

B

Mathematics gives the pupil a taste for precision, teaches him to base his conclusions on exact observations, and helps him to acquire a healthy critical spirit.

It inculcates good habits of thinking. Teachers should be meticulous not only for exactness of subject matter but also for correctness and elegance of form. This does not mean that the pupil should learn what he does not understand: he should be allowed some liberty of expression, so that if he makes mistakes these may be pointed out to him. This will foster the development of his judgment and his mastery of the language.

(Summarised from : Fédération Nationale de l'Enseignement Moyen Catholique, Programme et Directives 1961 — *Mathématiques*, p. 32 [5])

Having been initiated into mathematical language and symbolism in the early years, the pupil in the upper classes should acquire a spirit of synthesis and a taste for research. The years spent at school should furnish the pupil with the necessary mathematical tools both for higher education and for everyday life.

Brief historical notes should be given to indicate the stages of the development of theories. Mathematics is an essential tool in the experimental sciences, in particular physics. Applications which have a particular relevance for science should be stressed.

It is recommended that modern methods and language be introduced where this will facilitate study and strengthen comprehension of topics in the syllabus.

(*ibid.*, p. 39, summarised)

(2) DENMARK

Mathematics in the mathematics section

The objects of the course are to give the pupils a knowledge of mathematical thinking and of a number of fundamental mathematical concepts, to arouse their feeling for clarity and logical consistency in demonstration and expression, to develop their imagination and resourcefulness, to train them in dealing with concrete problems, including the making of numerical calculations, and to accustom them to the use of mathematics in other subjects.

(Extract from : *Guidance on the Curriculum in the Gymnasium* [11])

(3) FRANCE

A

Mathematics should impart a 'culture générale', foster the pupil's intelligence, character and moral sense and develop his mind through logic and precision. The pupil should be made to appreciate the potential and the limits of the mind and to see what it takes to bring a piece of work to a successful conclusion. Mathematics is important in the integral training of a child as it learns: (1) to separate the true from the false, (2) to observe and connect observations with principles, (3) to measure, and criticise facts, (4) to see complex questions in their entirety, to analyse and organise, (5) to express himself orally and in writing.

There is no need to scorn knowledge but more importance should be attached to the intellectual benefit that the child receives from his studies. *Esprit libéral* — mathematics is 'a good school of thought' and the pupil should learn not to judge things on the word of others alone nor to discuss matters about which he is ignorant. Each pupil should take an effective part in the work. The study and solution of problems shows the ability to apply knowledge.

The different disciplines should be interrelated, thereby leading to a more complete training of the mind. The pupil should have the ability to explain various aspects of a question in terms of other 'subjects' — historical aspect — other civilisations. Mathematics is a part of life and a reasonable knowledge of mathematics is therefore essential; in view of points 1-5 mathematics is an indispensable element of real culture.

(Summarised from: *Les Mathématiques*, Brochure No. 59 of Ministère de l'Education Nationale, I.P.N., 1964 pp. 5 *et seq.* [6].)

Classe de mathématiques élémentaires (Now Section C)

Recently an extension towards analytic geometry. vector analysis, as well as greater unity, in light of modern mathematics i. e. one unified syllabus, as far as possible. — Not fragmentation into traditional notions. The method of 're-discovery' is used.

(*School Mathematics in OEEC Countries*, p. 19 [1].)

(') FEDERAL REPUBLIC OF GERMANY

A

Functions of mathematics teaching

1. The teaching of mathematics should permit the pupil to experience how mathematical problems originate, how they are solved and how both problems and solutions can be arranged in an overall mathematical relationship.

2. The pupil should experience mathematics as an exact science, in that in mathematics he should be able to derive a wholly related area of facts from a few obvious principles. He should also realise that one can become critical or have reflections about the self-evidence of principles, fundamental problems, and their relation to philosophy.

3. He should recognise how simple experiment can give rise to mathematical reflection — the origin of Greek mathematics gives such examples — and how new problems, special concepts and methods car 'se from this.

4. The pupil should realise the by-no-means obvious applicability of mathematics to the exact sciences, and the limits of its possibilities.

5. The pupil should understand the efficiency of calculus but he should also realise that its automatic use may inhibit complete understanding.

(*Empfehlungen und Gutachten des Deutschen Ausschusses für das Erziehungs- und Bildungswesen*, Folge 9, p. 51 [7].)

B

The method of teaching becomes increasingly deductive (in the top grade) dealing with elements of an abstract character. The essential aims of mathematical instruction in the upper grade are: the power of abstraction, the foundations as well as the logical structure of mathematics, methods of scientific research and the discovery of cultural and historical connections. It is to be closely co-ordinated with physics teaching.

(*School Mathematics in OEEC Countries*, p. 28 [1].)

C

Tasks and aims of mathematics teaching

(*a*) Mathematics is the foundation of our culture and civilisation. The pupil is to appreciate mathematics as the most impressive example of an exact science. He should experience its meaning for the history of ideas and for the several aspects of reality.

(*b*) From chosen examples the teaching should give the pupil insight into the many beautiful relationships in numbers and figures, and the ability to recognise relationships of size and form also in the outside world must be developed.

(*c*) The pupil should learn how to solve problems in pure and applied mathematics; to know how to pick out the main points of a problem, to find suitable solutions, and to express them clearly.

(*d*) The pupil should acquire an understanding of mathematical working methods and proofs. He should realise the importance of mathematical knowledge but also its limitations. In some cases philosophical aspects of the question can be dealt with.

(*e*) The teaching of mathematics should develop the power of abstraction, an aptitude for critical thought, a capacity for synthesis and the power of geometrical representation. It should also develop exactness and thoroughness.

Teaching directives

(*a*) In order to facilitate the modernisation of mathematics teaching, much of the material commonly used up to now must be discarded. In its place, overall connections and central ideas, and logical and methodological viewpoints should be given most importance.

(*b*) The concepts of number, limit value, function vector, should maintain their central position in mathematics teaching in all classes.

(*c*) Mathematical methods of work and thought show clearly the place of mathematics in the intellectual world and in particular in academic secondary education.

(*d*) Mathematical thought is not only deductive; the discovery of propositions which have to be proved, and the possible methods of proof, intuitive knowledge, general and inductive as well as analogical conclusions are all equally important for successful work in mathematics.

(*e*) The role of mathematics in natural sciences, technology, art, sociology and economics can be shown by examples. The interest of many pupils in the subject can be awakened if reference is made to these applications of mathematics.

(*f*) In all grades some remarks on the history of problems and culture should show the importance of mathematics in the intellectual world. Such remarks have a general cultural value.

Twelfth and thirteenth school years

The teacher in the upper grades has extensive freedom to choose the subjects and methods necessary to give an idea of the modes of mathematical thought and work as well as of mathematical relationships. The choice of subject matter should depend on a basic theme leading necessarily to study in depth.

(Nordrhein-Westfalen, *Richtlinien für den Unterricht in der Höheren Schule: Mathematik*, p. 1 [8].)

D

Aim

Teaching should introduce students to an understanding and experience of mathematics as an intellectual structure, a system that in the clarity and general validity of its conclusions is a pattern or prototype for all the exact sciences, affording a means to describe and explain natural phenomena. Familiarity with numbers and forms should lead pupils to a mastery of the most important laws and methods in algebra and geometry and to an understanding of functional relations; they should study special concepts more fully and should introduce into their ideas and thinking more precision and logic.

Mathematics necessarily demands concise and correct form of expression and the pupil should be accustomed to thoroughness, objectivity and order. Teaching is successful when it gives the pupils a sense of achievement, joy in discovery, as well as a feeling for clarity and beauty in mathematics. Some philosophical aspects should be dealt with but the limitations of knowledge should also be made evident. The teaching should be based on facts when natural phenomena and technical matters are being dealt with.

Teaching methods

The inductive method is used, but where possible and suitable the deductive method should also be employed. The pupil must have the opportunity for independent and critical thinking; questions should be asked and solutions suggested. The stress should be on understanding, on concise, clear expression, and on the translation of words to symbols and vice versa. The basic concepts of statistics and the calculation of possibility should be considered. There should be a link with reality and with other subjects e. g. problems of possibility, chance, life insurance.

(*Kultusministerium*, Baden-Württemberg: *Mathematik*, p. 180 [9].)

(5) LUXEMBOURG

Mathematics methodology

The aim is to contribute to the education of the pupil by developing in him a feeling for logic, intuition, and a spirit of research. Besides this it provides him with a sound knowledge, indispensable in view of the important role that mathematics plays in modern civilisation.

In this branch of knowledge more than in any other, learning means understanding and it is therefore important that all abstract notions are understood; it is necessary to introduce them via the concrete. As a general rule the method of exposition must be experimental; this should make the teaching more lively and active. The precision of mathematics demands that the language should be simple and accurate.

Classes supérieures

The separation into scientific and literary sections allows greater depth of knowledge to be imparted in *section latine B*, which is the science branch. Subject-matter which will facilitate entry to university is dealt with. The course should lead to a true mathematical culture. More time is devoted to reasoning (discussion of problems), the pupil's appreciation of space is developed, and an attempt is made to give them an overall view. Despite the apparent diversity of disciplines one should make the pupils see that there is only *one* mathematical science by emphasising the many connections between algebra, geometry and trigonometry. Opportunities for this abound: graphic representation and graphic determination of the systems of equations. It should be shown that it is possible to group the topics of the course around some central ideas, such as function, limit values, geometric transformation. An extension of the notion of number, and revision of work before the study of logarithms affords an opportunity for a profitable synthesis. Time should be found to discuss the evolution of mathematics, to give historical detail, and the application of mathematics to the fields of art and technology should be cited.

(Extract from: *Enseignement Secondaire et Supérieur:* Institut Pédagogique - Luxembourg. 1962-63. p. 54 [12].)

(6) SWEDEN

Goal

The aim of mathematics teaching is:

(1) to acquaint pupils with certain essential concepts and methods in algebra, geometry, the theory of functions, probability theory and statistics;

(2) to make them proficient in numerical calculation, particularly with the help of technical aids;

(3) to give an insight into the uses of mathematics in other subjects.

Directives and comments

The teaching of mathematics should be based on the assumption that only a few of the pupils will later take up the study of mathematics as a science. For most people mathematics is an instrument that is needed for further studies or in one's profession. It is therefore important that the teaching should illustrate the application of mathematics within other fields. It is natural that the applications should be taken from economics and social science in the Social Sciences and Economics Course and from science and technology in the Natural Sciences and Technology Course.

The mathematics course revolves basically around the headings of algebra, geometry, theory of functions, probability theory and statistics. There should be no strict delineation, howewer, and the mathematics course should be integrated as far as possible.

Concepts and symbols from the theory of sets should be used, e. g. for introducing the general concept of functions, as in probability theory. There should not be a formal course on the theory of sets with associated symbolics, but pupils should become successively aquainted with the manner of thinking of the theory of sets. The essential point should be the methodics of the theory of sets, not the symbolics.

The teaching should include the historical aspects of mathematics, e. g. the contributions of Archimedes and Newton in respect of differential and integral calculus. The history of probability theory can also be presented.

Numerical methods have acquired a great deal of importance through the rapid development and increased application of computers. This should receive attention within different sections of the course. Numerical methods of solving equations, numerical calculation of integrals, and the fitting of polynomials to given co-ordinates should be gone through. Attention should be paid to the possibility of illustrating calculations by means of flow charts.

The slide rule and calculator should be used in numerical calculations. The pupil's ability to do rough calculations should be trained. One of the objectives should be to exercise proficiency in numerical calculation.

The teaching of mathematics should accustom pupils to a lucid and exact mode of expression in the presentation of proofs and logical reasoning, for mathematics — more that most subjects — offers opportunities for analysis and logical argument and for the discussion of the significance of well-defined concepts.

(Extract from notes kindly supplied by the Royal Swedish Ministry of Education.)

III. EDUCATIONAL OBJECTIVES AND EXAMINING IN MATHEMATICS

The concept of a taxonomy of intellectual processes

In any examination, whether intended as a secondary school leaving diploma or as an admission qualification to university. we must look not only for a certain minimum amount of knowledge in each candidate before he seeks entrance to the university. but for something more. Any valid and reliable examination at this level should test a variety of intellectual processes. The first is obviously the degree of knowledge that the candidate possesses of the syllabus. Knowledge itself may be divided into a number of subsections. such as the ability to use terminology of any particular subject, the ability to deal in specific facts related to that subject and the knowledge of principles and generalisations which occur in the syllabus. Besides the acquisition of knowledge, however, we should also try to encourage in the future university student certain attitndes, certain modes of thinking. methods of approaching problems, and ways of criticising and evaluating. An analysis of such educational objectives has been termed by its compiler (Dr. Bloom of Chicago) a 'taxonomy'. (A condensed version of this taxonomy is given in Appendix 2.) In this part of his taxonomy Bloom is concerned only with the cognitive domain of educational objectives, which represent the intellectual processes that it is hoped that the syllabuses are instrumental in inculcating; for present purposes a consideration of the cognitive domain will suffice.

Bloom has classified his taxonomy into six main categories, arranged in hierarchical order. There is firstly knowledge, which is divided into a considerable number of sub-sections; then comprehension which is further defined into subsections; there follow application. analysis, synthesis and evaluation, which are also usually sub-divided. Some examples of these sections and what is meant by them are given below. The first part of comprehension, for instance. is defined as 'translation'. In mathematics this might be the ability to translate the problem which is presented into technical or abstract phraseology into concrete or less abstract phraseology. Alternatively, it might be translation from one symbolic form into another. The second part of comprehension is interpretation. which might be the distinguishing of true and logical conclusions from false and illogical conclusions drawn from a body of data. Thirdly, there is extrapolation : this requires not only the ability to translate and interpret facts. but the capacity to extend trends or tendencies beyond those facts, considering what are the implications and consequences from them.

At an international meeting of mathematics experts convened in Strasbourg in connection with the Oxford-Council of Europe Evaluation Study, Bloom's taxonomy of intellectual processes was used as the basis to derive a particular taxonomy for mathematics at the academic secondary level. Using Bloom's work as a guide the taxonomy given below was evolved:

Taxonomy of educational objectives for mathematics

A. KNOWLEDGE

A1 *Knowledge of language and facts*
- A11 Knowledge of terminology
- A12 Knowledge of conventions
- A13 Knowledge of classifications
- A14 Knowledge of structnres

A2 *Knowledge of avenues of access to results*
- A21 Knowledge of methodology
- A22 Knowledge of principles and generalisations
- A23 Knowledge of theories

A3 *Knowledge of results*
A31 Knowledge of specific facts
A32 Knowledge of criteria

B. COMPREHENSION

B1 Recognising the nature of a problem
B2 Understanding the relation between cause and effect
B3 Ability to follow a logical sequence

C. TRANSLATION

C1 Expression of data in a different form as evidence of comprehension
C2 Expressing clearly an intricate line of reasoning

D. APPLICATION

D1 Technical and formal application
D2 As further evidence of comprehension to induce the student to apply his knowledge to new situations

E. INVENTIVENESS

Ability to go beyond what has been taught

An example of the way in which such a taxonomy can be applied in examining is given in Table II [1]. A 'weighting' is assigned to each section of the taxonomy. The questions set are then distributed so as to comply with the suggested weighting, so that all the diverse elements of the taxonomy are adequately — but not necessarily evenly — covered by a judicious 'sampling' of the syllabus. In the example given below the question paper was divided into two sections:

Section A · 35 multiple-choice questions, each of which were allotted 1 or 2 marks;

Section B · questions of the more traditional 'essay' type or problem kind, in which the range of marks was from 2 to 8.

Table II shows the *taxonomic value* of each question, i. e. the point of classification (see the *Taxonomy of educational objectives for mathematics* given above) that the questions set actually cover. The addition horizontally across the columns shows the total mark per question. The addition vertically down the columns shows the weighting assigned to each group in the taxonomy. It will be noted that in Section A of the question paper (which cannot be given here because it has still to be worked by candidates in an international examination) the emphasis is on knowledge (A). On the other hand, in the question paper as a whole the overall emphasis lies on application (D), as the *use* of acquired knowledge, at the particular level for which the question paper was devised (the top classes of the academic secondary schools), was considered of greater importance than the knowledge itself. It can also be seen that at this level little weight was given to inventiveness (E).

The advantages of this more systematic approach to the problems of examining in mathematics are obvious: chief among them is the fact that it is possible to evaluate more precisely how far the aims and objectives propounded are in fact being achieved.

1. The example is that of an international mathematics test devised in connection with the Oxford · Council of Europe Evaluation Study. The rationale and results of this test are to be published in late 1968. For help in the preparation of the taxonomy and its application to testing we should like to express our gratitude to Mr. David Tinsley, St. Edward's School, Oxford.

TABLE II

Taxonomy of a proposed international mathematics paper

Taxonomy Group Question No.	A			B			C		D		E	Mark per Question
	1	2	3	1	2	3	1	2	1	2		
A 1	1											1
2			1									1
3			1									1
4			1									1
5	1											1
6			1/2				1/2					1
7			1/2						1/2			1
8			1									1
9	1											1
10	1											1
11			1									1
12									1			1
13									1			1
14									1			1
15							1/2		1/2			1
16									1			1
17									1			1
18									1			1
19									1			1
20	1											1
21					1							1
22							1					1
23	1											1
24			1									1
25			1									1
26			1									1
27			1									1
28			1									1
29			1/2				1/2					1
30			1/2						1/2			1
31			1				1/2		1/2			2
32							2					2
33			1						1			2
34			1				1/2			1/2		2
35					2							2
TOTAL	6	—	15	—	3	—	5 1/2	—	10	1/2	—	40

Taxonomy Group Question No.	A			B			C		D		E	Mark per Question
	1	2	3	1	2	3	1	2	1	2		
B 1										2		2
2				2								2
3						2						2
4			1					1				2
5									3			3
6									3			3
7					1					2		[illegible]
8							4					4
9									4			4
10								2			2	4
11						4						4
12				1			1		2			4
13				2				2				4
14					2					3		5
15									2	4		6
16									8			8
B TOTAL	—	—	1	5	3	6	5	5	22	11	2	60
A TOTAL	6	—	15	—	3	--	$5^{1}/_{2}$	—	10	$^{1}/_{2}$	—	40
PAPER I TOTALS	6	—	16	5	6	6	$10^{1}/_{2}$	5	32	$11^{1}/_{2}$	2	100

IV. DETERMINATION OF THE SYLLABUS

There are four principal ways in which countries determine the mathematics syllabuses for the academic secondary school. In the majority of countries the syllabuses are usually determined centrally, usually within the Ministry of Education, but also with the consultation of mathematics experts, who may be inspectors, as in France, or teachers, as in Belgium, or both, as in Denmark. Moreover, everywhere 'pressure groups' — bodies of teachers or professional associations — operate by means of publications, meetings etc. to promote a climate for reform. A particular case is that of the Federal Republic of Germany which, under its Federal constitution, has a Ministry of Education in each *Land*. The syllabuses are therefore determined at the level of the *Land*, but otherwise the procedure followed is as above. Moreover, nation-wide teachers' associations and publications have the effect of producing a broad consensus as to content. Yet another special case is that of Switzerland, where each of the 27 cantons has its own department of education : here the process of syllabus determination is that of teacher consultation and of recommendations by them to the cantonal authority. To a lesser degree the Netherlands also represents a special case, since from a number of topics the Ministry selects each year those which shall be studied. The most exceptional procedure is that followed in England, where syllabuses are determined by eight examining boards, which consult both university and school teachers. These boards (with one exception) are nominally under the aegis of a university or a group of universities. Schools may affiliate themselves to any board, which also has the power to refuse an application for affiliation. Whilst syllabuses vary, the divergences are not all that great between them. In all countries, of course, the demands of further and higher education courses will directly or indirectly influence syllabus content. The overall pattern of syllabus determination would therefore seem to be one where the task is in the hands of the specialists — inspectors, university and school teachers — and is carried out under the supervision of central administrators.

Selection of textbooks

The simplest procedure — and the most usual — is for the mathematics teachers, with the concurrence of the headmaster, to make their own choice of textbooks from a list approved by the central Ministry. This list itself has usually been compiled by a committee of mathematics experts composed of inspectors and outstanding teachers. In England the individual teacher, subject to the headmaster's approval, has complete liberty of choice, since no national standards are laid down and no recommended list is published. At the opposite end of the spectrum, Luxembourg deems it necessary to prescribe in detail the textbooks to be used. What kind of choice, if any, should be allowed the teacher is in any case debatable: the younger teacher, given complete freedom of choice, may be confused by the sheer *embarras du choix* ; the older teacher, given no choice, may find himself compelled to use a textbook that he dislikes intensely.

V. STRUCTURE OF THE SYLLABUS

1. Syllabus analysis

Since mathematics uses symbolic language, it has, in this respect, a remarkably international character. A study of the organisation and content of the various mathematics curricula reveals, however, great difference of opinion as to the amount of time to be allotted to mathematics, its place in the curriculum as a whole, the objectives of mathematics teaching, and the content of the syllabus. A topic which is thought to be important in one country and would therefore be studied in great detail and to some considerable depth, may be merely touched upon or even completely excluded from the syllabus by another country.

The syllabuses in mathematics at the most specialised level were considered for some 15 countries and an item analysis was made, so that a complete classification of the overall content of *all* these syllabuses was evolved [1]. The classification was based on a modification of the Universal Decimal Classification System. This detailed analysis contained over 180 items, and was considered by representatives from each of the countries. They indicated whether or not each item in the analysis was studied by the most specialised mathematics student in the academic secondary school in their country. A study of the analysis revealed only a very small number of items which were common to all of the countries (" common core "). This lack of consensus between countries is all the more surprising when one considers that it is normally from the most specialised mathematics sections in the secondary school that are drawn the specialist university students in mathematics. In view of this, it would be interesting to investigate whether more uniformity regarding the syllabuses exists at university level than apparently does at the secondary one. The common topics are listed in Table III and the depth to which the topic is studied is also indicated.

The common core topics can be expanded if we also include topics which were common to all but two countries. These topics are given in Table IV.

2. Differing emphases in the syllabus

In order to have a *very approximate* idea of the emphasis that each country places on the various branches of mathematics in percentage form, the item analysis of the syllabus was used but here the items were grouped under seven traditional headings, which were as follows:

1. Algebra
2. Abstract algebra
3. Euclidean geometry
4. Trigonometry
5. Descriptive geometry
6. Calculus
7. Probability and Statistics

(Arithmetic, the history of mathematics, kinematics and theoretical mechanics, which usually form a small part of the syllabus analysis, were omitted when the following tables were compiled.)

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1. See Appendices III and XII.

TABLE III

Common core topics

x	Depth 1 - the topic is studied on a superficial level
o	Depth 2 - the topic is studied to a moderate degree
	Depth 3 - the topic is studied to a very great degree

The number of squares indicates the number of countries.

Code	Topic																
512.02	Polynomials								o	o	o	o	o	o	o	o	o
512.03 512.04	Rational Functions and their Graphs							o	o	o	o	o	o	o	o	o	x
512.07	Relations between roots of P(2) = O 2nd degree								o	o	o	o	o	o	o	o	o
512.21	Algebraic equations									o	o	o	o	o	o	x	x
513.31	Mensuration of pyramid. cone. sphere (volume and surface)							o	o	o	o	o	o	o	x	x	x
514.51	Sine and cosine rule for solutions of triangles									o	o	o	o	o	x	x	x
516.11	Analytical geometry of straight line and circle										o	o	o	o	o	o	x
516.21	Focus and directrix property of parabola. ellipse. hyperbola												o	o	o	o	o
516.23	Analytical geometry of parabola							o	o	o	o	o	o	o	o	o	x
516.24	Analytical geometry of ellipse, hyperbola referred to principal axes and rectangular hyperbola to assymptotes						o	o	o	o	o	o	o	o	o	x	x
517.200	Differentiation and derivatives							o	o	o	o	o	o	o	o	x	x

TABLE IV

Additional common core topics

512.05	Identities							o	o	o	o	o	o	o		x	
512.08	Inequalities involving modification. squaring, square root, 2nd degree							o	o	o	o	o	o	¦	x	x	
512.81	Natural numbers (theory of numbers)	o	o	o	o	o	o	o	o	o	¦	x	x	x	x		
517.11	Limit F(n) $n \to \infty$		o	o	o	o	o	o	o	¦		x	x	x	x	x	
517.14	Limit F(x) $x \to a$		o	o	o	o	o	o	o	o	x	x	x	x	x	x	
517.15	Continuity	o	o	o	o	o	o	¦	x	x	x	x	x	x	x		
517.215	Arithmetic, geometric series						o	o	o	o	o	o	o	o	x		
517.273	Maxima, minima, inflexions						o	o	o	o	o	o	o	o	o	x	
517.30	Definition and properties of definite integral		o	o	o	o	o	o	o	o	o	¦	x	x	x		
512.821	Simultaneous equations (3 linears, 1 linear) 2nd degree					o	o	o	o	o	o	o	o	o	o	x	¦

¦ Indicates that information regarding 'depth' was insufficient.

Table V shows the emphases on the various sections of the syllabus by country. The 'total' syllabus for each country is regarded as 100 per cent and each of the seven branches is expressed as a percentage of this total syllabus. From this table it can be seen where the main emphasis would *appear* to lie in each country; the majority of the countries seem to place the greatest emphasis on calculus (no less than 12) and the least emphasis on trigonometry or statistics (6 countries do not study statistics at all) [1].

1. For a list of topics which are common to each country see Appendix IV.

TABLE V

Emphases placed on the various sections of the syllabus by country (expressed as percentages)

Austria	
Calculus	32
Des. geo.	19
Algebra	17
Abs. alge.	15
Trig.	9
Euc. geo.	6
Stats.	2
	± 100

Belgium	
Calculus	26
Des. geo.	23
Algebra	18
Euc. geo.	18
Abs. alge.	11
Trig.	5
Stats.	2
	± 100

Cyprus	
Abs. alge.	39
Calculus	26
Algebra	17
Des. geo	7
Euc. geo.	4
Stats.	4
Trig.	1
	± 100

Fed. Rep. of Germany	
Algebra	30
Des. geo.	25
Abs. alge.	20
Calculus	15
Euc. geo.	5
Trig.	5
Stats.	0
	± 100

France	
Abs. alge.	29
Calculus	24
Des. geo.	19
Algebra	16
Euc. geo.	9
Trig.	3
Stats.	0
	± 100

Greece	
Abs. alge.	37
Calculus	34
Algebra	14
Des. geo.	7
Euc. geo.	4
Stats.	3
Trig.	1
	± 100

Luxembourg	
Calculus	33
Abs. alge.	28
Algebra	16
Des. geo.	15
Euc. geo.	4
Stats.	3
Trig.	1
	± 100

Ireland	
Calculus	30
Abs. alge.	22
Des. geo.	17
Algebra	14
Stats.	9
Euc. geo.	7
Trig.	1
	± 100

Italy	
Calculus	36
Abs. alge.	23
Algebra	20
Des. geo.	11
Euc. geo.	7
Stats.	2
Trig.	0
	± 100

Netherlands	
Calculus	30
Abs. alge.	22
Algebra	19
Des. geo.	19
Euc. geo.	8
Trig.	3
Stats.	0
	± 100

Norway	
Calculus	34
Abs. alge.	29
Algebra	21
Des. geo.	11
Euc. geo.	3
Trig.	3
Stats.	0
	± 100

Sweden	
Calculus	39
Abs. alge.	23
Algebra	13
Stats.	13
Des. geo.	10
Euc. geo.	1
Trig.	1
	± 100

Switzerland	
Calculus	36
Abs. alge.	30
Algebra	12
Des. geo.	13
Euc. geo.	3
Trig.	3
Stats.	1
	± 100

Turkey	
Calculus	35
Des. geo.	24
Algebra	15
Abs. alge.	12
Euc. geo.	12
Trig.	2
Stats.	0
	± 100

United Kingdom	
Calculus	42
Des. geo.	17
Stats.	16
Algebra	12
Abs. alge.	9
Euc. geo.	4
Trig.	1
	± 100

Finland	
Calculus	35
Algebra	20
Abs. alge.	15
Des. geo.	15
Euc. geo.	12.5
Trig.	2.5
Stats.	0
	± 100

Table VI shows the percentage of each branch of the syllabus that each country appears to cover. The various branches are represented as 100 per cent and the amount that each country deals with is expressed as a percentage of this. e. g. Austria would cover 69 per cent of the algebra in the total syllabus analysis, but only 6 per cent of the probability and statistics syllabus. From this table we can see that the percentage range within the sections of the syllabus is extensive, e. g. the percentage of the calculus syllabus deals with ranges from 6 to 78 per cent and the abstract algebra range is from 10 per cent to 72 per cent.

The following table gives some indication of the "completeness" of the syllabus coverage, by branches, in each country.

TABLE VI

Percentage of the total number of topics dealt with by each country in the various parts of the syllabus

	Au	Bg	Cy	Fr	FRG	Gr	Lu	Ir
Algebra	69	92	92	92	46	85	92	85
Abstract Algebra	21	18	69	56	10	72	54	46
Euclidean Geometry	23	92	23	54	8	23	23	46
Trigonom.	63	38	13	25	13	13	13	13
Descriptive Geometry	40	60	20	56	20	20	44	56
Calculus	31	31	33	33	6	48	46	44
Probability Statistics	6	0	18	0	0	12	12	41

	It	Ne	No	Sd	Sw	Tu	UK	Fi
Algebra	69	54	62	77	85	77	92	62
Abstract Algebra	26	21	28	47	69	21	23	15
Euclidean Geometry	23	23	8	8	23	62	31	38
Trigonom.	0	13	13	13	38	13	13	13
Descriptive Geometry	20	28	16	32	48	64	68	24
Calculus	30	20	24	57	59	43	78	26
Probability Statistics	6	0	0	59	6	0	94	0

The total number of items contained in the seven sections under consideration was over 180, and it was found that the United Kingdom covered the largest percentage of the items (approximately 60 per cent). This is to be expected as England (the largest constituent unit of Great Britain) is the only country which has pre-university specialisation to such an extent. There the mathematics student in the Sixth Form (the top grade) of the academic secondary school will usually be studying only science subjects, probably physics and chemistry in addition to mathematics, whereas the Continental student carries five or six subjects of a much more general nature [1]. United Kingdom. however, does not spend the greatest amount of time on mathematics, as will be seen from the table which appears later in this section (Table VIII). United Kingdom devotes about 373 hours to mathematics in the two years preceding the terminal examination, whereas Luxembourg in its most specialised section spends some 480 hours on mathematics and yet deals with a much smaller percentage of the sum total of the syllabuses (about 43 per cent). The explanation may well be that study in depth of fewer topics is considered pedagogically of greater value.

Table VII gives an indication -- and it can be no more -- of the percentage of teaching time devoted in each country to each of the seven main divisions of the syllabus.

TABLE VII

Percentage of total mathematics teaching time spent on the 7 sections of the syllabus in each country

	Austria	Belgium	Cyprus	France	Fed. Rep. of Germany	Greece
Algebra	17	18	17	16	30	14
Abstract Algebra	15	11	39	29	20	37
Euclidean Geometry	6	18	4	9	5	4
Trigonometry	9	5	1	3	5	1
Descriptive Geometry	19	23	7	19	25	7
Calculus	32	26	26	24	15	34
Probability Statistics	2	2	4	0	0	3
Total ± 100 %						

	Luxembourg	Ireland	Italy	Netherlands	Norway	Sweden
Algebra	16	14	20	19	21	13
Abstract Algebra	28	22	23	22	29	23
Euclidean Geometry	4	7	7	8	3	1
Trigonometry	1	1	0	3	3	1
Descriptive Geometry	15	17	11	19	11	10
Calculus	33	30	36	30	34	39
Probability Statistics	3	9	2	0	0	13
Total ± 100 %						

1. In England, however, the degree of specialisation may be even greater. The Sixth Former can study only three subjects, two of them being Pure and Applied Mathematics and the other usually Physics. In this case as much as 45 per cent of the student's time may be devoted to the study of mathematics alone.

	Switzer-land	Turkey	United Kingdom	Finland
Algebra	12	15	12	20
Abstract Algebra	30	12	9	15
Euclidean Geometry	3		4	13
Trigonometry	3	2	1	3
Descriptive Geometry	13	24	17	15
Calculus	36	35	42	35
Probability Statistics	1	0	16	0
Total ± 100 %				

3. Allocation of teaching time

An approximate calculation was made of the amount of time devoted to mathematics in the most specialised section of the academic secondary schools calculated over the last two years, as shown by the information given in Appendix 5.

As can be seen from the table given below, there seems to be considerable variation, with Luxembourg devoting some 480 hours to mathematics study in the last two years and Greece, by contrast, 200 hours. The median was about 296 hours and the mean number of hours was about 311 hours.

TABLE VIII

Amount of time devoted to mathematics in the most specialised section of the most specialised academic secondary school in the last two years [1].
(To nearest hour of 60 minutes)

1. Luxembourg	480
2. France	455
3. Belgium	408
4. United Kingdom	373 [2]
5. Switzerland	360
6. Norway	325
7. Netherlands	321
8. Turkey	312
9. Germany (Nordrhein-Westfalen)	280
10. Sweden	276
11. Ireland	256
12. Finland	248
13. Cyprus	243
14. Austria	240
15. Italy	204
16. Greece	200

A comparison was also made of the time devoted to mathematics in various countries in relation to other subjects studied. This is shown in Table IX.

1. But see Note 1 on page 31.
2. The information in Table VIII was gained from a questionaire which was answered by representatives from each of the countries involved in the survey. The questionaire is given in Appendix 5.

TABLE IX

Number of lessons per week devoted to mathematics in the most specialised section of the academic secondary school in the last year

(selected countries only)

	Bg[1]	Bg[2]	De	FRG[1]	FRG[2]	It	Lu	Ne[1]	Ne[2]	No	Sd	U.K.
Mathematics	9	9	6	4	4	3	6	5	5	7	5	7
Physics	2	2	4	3	6	3	3	3	3	6	3.5	7
Chemistry			2	3		2	2	4	4		2.5	7
Biology			4					2	2		2	
Other Sciences				1[c]				2[a]		1[b]		
Other Subjects	20	17	20	18	19	22	20	18	19	22	19	7
Total	31	28	36	29	29	30	31	34	33	36	32	28*

The duration of the lesson period varies from country to country from 40 minutes to 1 hour [1].

Bg[1] Belginm Catholic - *Humanités anciennes*, Latin-maths branch

Bg[2] Belgium Catholic - *Humanités modernes*, Science A branch

FRG[1] *Land* Baden-Württemberg

FRG[2] *Land* Hessen

Ne[1] *Hogereburgerschool* - Division B

Nc[2] *Gymnasium* - Division B

* *Private study periods in mathematics and the sciences* are allowed, making the total school load some 35 periods a week. (See also Note 1, page 31).

a Mechanic

b Descriptive geometry

c Practical

The above table is shown below in percentage form. With the exception of Belgium (which, it will be recalled, has only a six-year secondary course) the United Kingdom (which has a 7-8 year course) devotes the greatest percentage of time to mathematics, as was to be expected in view of its principle of specialisation ; likewise, it devotes the least percentage of time to non-mathematical and non-scientific subjects. Italy, with its great humanist heritage, obviously believes that its future mathematicians and scientists should be very well versed in the humanities.

1. Sample timetables from selected countries are given in Appendix VI.

TABLE X

Percentage of time devoted to mathematics in relation to other subjects
(selected countries only)

	Bg[1]	Bg[2]	De	FRG[1]	FRG[2]	It	Lu	Ne[1]	Ne[2]	No	Sd	U.K.
Mathematics	30	32	17	14	14	10	19	15	15	19	16	25
Physics			11	10		10	10	9	9	17	11	25
Chemistry	6	7	6	10	20	7	6	12	12		9	25
Biology			11					6	6		9	
Other Sciences				3				6		3		
Arts etc.	64	61	55	62	65	73	64	53	58	61	59	25
Total ±	100	100	100	100	100	100	100	100	100	100	100	100

The following table which shows the percentage of time devoted to mathematics and scientific subjects, as opposed to all other elements in the curriculum — mainly the humanities — may be of interest as illustrating different approaches to the education of the gifted student.

TABLE XI

Percentage of time devoted to mathematics and science in relation to other subjects
(selected countries only)

	Bg[1]	Bg[2]	De	FRG[1]	FRG[2]	It	Lu	Ne[1]	Ne[2]	No	Sd	U.K.
Mathematics and Science	36	39	45	37	34	27	35	42	42	39	42	75
Other Subjects	64	61	55	62	65	73	64	53	58	61	59	25
Total ±	100	100	100	100	100	100	100	100	100	10′	100	100

The duration of the mathematics syllabus in preparation for the terminal examination varies considerably from country to country. Most countries state that their examination is based on the last two years (or three years) of the academic secondary school, but some countries assert categorically that the questions are drawn from the whole of the syllabus studied throughout the student's school career. In practice, this does not seem to make much difference. as even with those countries who state that they draw their questions only from the formal programme of the last two or three years, in point of fact previous knowledge is always assumed. The only difficulty that arises is that, with topics treated at an earlier stage in the school career. the degree of depth to which they are carried will of course be much less.

VI. FUTURE TRENDS IN THE SYLLABUS

The summary given below shows the replies given by mathematics experts from various European countries when they were asked for a brief description of any changes contemplated in their syllabus in the near future. So far as the introduction of modern mathematics is concerned, there seems to be no universal policy. Some countries have introduced it in the primary or the lower secondary school, and propose to continue it right through the secondary school (e.g. Austria, Belgium and United Kingdom, with its various local projects). Greece contemplates no change at all, but it will be remembered that Greece has in fact just reformed its academic certificate of secondary education.

Sufficiently detailed information was secured from eight of the countries in the survey for an item analysis table of the changes in the syllabus to be compiled (see Appendix 7). It shows the modifications contemplated during the next two to eight years and shows, where possible. the number of years ahead from 1966 when the changes are to be introduced. If the topic which one country plans to introduce into the syllabus is already studied elsewhere a cross is indicated against the topic. It will be seen that many of the topics shortly to be included in the syllabus are concerned with modern mathematics; there is a concentration upon set theory and vectors. Some countries are also beginning to introduce probability theory and statistics.

Among the many mathematics projects in train in European countries is the *Nuffield Foundation Mathematics Teaching Project* which may be cited as typical. This project is concerned not only with the content of the syllabus, i.e. a changeover to topics such as sets, vector spaces. statistics and computers, but also with method. The method revolution with its emphasis on learning, discovery and genuine understanding is considered even more important than the content. Throughout the United Kingdom 91 trial areas have been officially involved in experiments and so far guides for teachers of children up to the age of about 9 have been written, but the project is not committed too closely as to when children 'ought' to have reached a given point.

Summary of proposed changes in mathematics syllabuses 1967 — 1974

AUSTRIA

New syllabus will be in force next year for pupils aged 15 (5th form of the secondary school). Under the new regulations pupils will have to attend secondary school for 9 years instead of 8 as they do now. The new syllabus will be valid for all classes of the secondary school, especially the terminal year, in 5 years' time. In the new syllabus topics of modern mathematics will be introduced (theory of sets, vectors etc.).

BELGIUM

Since 1961 there have been attempts and experiments at introducing modern mathematics into the syllabus. At the moment an experimental programme is taught in the lower classes of the *Athénées*. One of the sponsors is the *Centre Belge de Pédagogie de la mathématique.*

CYPRUS

It is expected that as from 1967 the new syllabus for the top grades, following the educational reform, will be completed. It is also expected that the mathematics syllabus in the upper secondary school will be further revised to include the recommendations of OECD in respect to modern mathematics.

With regard to the final examination, a proposal is being considered to substitute for the present system a uniform external examination set by the Ministry of Education in closer collaboration with practising teachers and organisations. Also being considered are proposals to introduce the examinations for the Academic Certificate as provided by the recent educational reform in Greece (this certificate is comparable to the *Baccalauréat Abitur*, etc., and satisfies entrance requirements to universities and colleges in Greece).

Examinations for science subjects in this certificate were held for the first time in Cyprus last September, so Cypriot students do not now have to travel to Greece for these.

FRANCE

Some modern mathematics is being introduced. The Ministry of Education is to appoint a Commission to elaborate systematically new syllabuses or, more exactly, to study the place modern mathematics will have in the curriculum.

FEDERAL REPUBLIC OF GERMANY
(Nordrhein-Westfalen)

The present *Richtlinien* [8] came into force in 1963. Modifications will probably be necessary due to the change of the beginning of the school year from Easter to autumn.

GREECE

None.

IRELAND

The syllabus is due for revision in 3 years' time.

ITALY

There is a general revision of all syllabuses of all subjects going on at the moment with a view to making them more modern and more formative.

LUXEMBOURG

For 1966/67 the syllabus for the terminal class will see the partial replacement of the synthetic study of transformations (translation, similitude, inversion) by an introduction to matrical calculus and the analytical study of equations of transformations.

In the modern section the study of descriptive geometry has been reduced (polyhedras, curved surfaces) in favour of analytical geometry.

NETHERLANDS

In 1968 a new school orgauisation comes into force. Perhaps it will be possible at the same time to change the mathematics syllabus in such a way as to introduce into it modern mathematics.

NORWAY

Introduction of modern mathematics.

SWITZERLAND

Modernisation of the syllabus and teaching in some of the cantons (Geneva, Vaud, Neuchatel, Berne).

SWEDEN

The new course is very different from the old one. New topics include theory of sets, vectors, computers, complex numbers (extended), differential equations, vector functions, three dimensional vectors, descriptive statistics. combinatories. probability theory and statistics.

TURKEY

The Ministry of Education has the modernisation of the mathematics curricula in mind. Modern mathematics is taught in the science *lycée* (for talented children). Summer courses are organised to train teachers in the new curriculum. A pilot *lycée* in Ankara and the Gazi teacher training college are trying out a suitably amended version of the science-*lycée* curricula.

UNITED KINGDOM

Most of the Examination Boards are contemplating the introduction of optional A level syllabus which will embody concepts of modern mathematics.

(*a*) School Mathematics Project (SMP) has been in operation for a pilot group of schools for one year and is a prototype of these syllabuses. It is anticipated that other syllabuses will retain its essential principles.

(*b*) Mathematics in Education and Industry Project (MEI) has an experimental syllabus similar to (*a*) with more emphasis on Applied Mathematics (including statistics).

(*c*) The Midlands Mathematics Experiment is producing a syllabus which is expected to contain the same essential features as (*a*).

FINLAND

Since 1960 Finland has taken part in Scandinavian co-operation for the modernisation of the curriculum in the teaching of mathematics. In connection with this, systematic experiments have taken place in about 20 secondary schools where, in the top three grades, they have allocated 6 weekly periods for the teaching of mathematics. In experimenting with new teaching material, special attention has been paid to the teaching of calculus and analysis.

When the Scandinavian committee has finished its work in 1967 it will suggest somewhat radical reforms in the teaching of mathematics at all levels of education.

VII. SOME COMMENTS ON TEACHING METHOD

Few countries lay down in detail the teaching method that should be followed. Where indications are given they are contained in official directives; as well as these, most countries have semi-official publications or publications by professional associations, which are widely used by teachers [1]. It is interesting that Finland does not confine itself to Finnish or Swedish publications on methods: teachers are also recommended to consult corresponding publications in German and in English (both from the United Kingdom and the U.S.A.). This would seem to be an idea worthy of adoption elsewhere. Some interesting ideas on teaching, from a few countries, are summarised below.

Only Sweden seems to prescribe a definite order in which the topics of the syllabus should be taught. It will be recalled that Sweden now has no academic terminal examination, but instead undertakes continuous assessment by means of tests. A prescribed order is therefore necessary so that the tests made at any stage cover in fact what the pupil has already been taught. Other countries, whilst not prescribing an order of teaching, indicate how much of the syllabus should be covered during the school year.

The tendency is everywhere towards the promotion of active methods. The French express it as follows:

"The active method is important — the retention of a certain number of ideas and facts which have not always been fully understood does not contribute to the pupil's intellectual training. Every pupil must participate effectively in the lesson. Much of the activity of the pupil should be devoted to the study and search for the solution of problems... In this way the ability of each pupil to apply what he has learned can be effectively followed; the development of this ability should be one of the main objectives of a 'cultural education'."

(Ministère de l'Education Nationale: *Les Mathématiques*, Brochure No. 59. I.P.N., 1964 [6])

The Belgian Ministry (Ministère de l'Instruction Publique, *Les Mathématiques*, 1955, pp. 5-6) likewise stresses that every new theory introduced should be illustrated by applications. The teacher should avoid immediate abstraction, but seek to give new ideas a concrete basis. He should show the pupils the principal stages in the search for the solution of a problem, so that they appreciate that such solutions are not arrived at by mere chance but are the result of a logical, intellectual operation. If various ways exist to solve a problem, these should be compared for elegance and for efficiency. Every opportunity should be taken to inculcate ideas of symmetry and analogy: thus the relationship between similar theories should be pointed out. The aim must be to group the facts around fundamental ideas, so as to give them a unity.

The concept of fundamental ideas or 'themes' is central also to German teaching method. Demanding little pre-knowledge, embracing as large an area of mathematics as possible, these themes should be conceptually and philosophically meaningful. Reference has already been made also to the German theory of *exemplarisches Lernen*. The teacher is urged to have his pupils study a few problems thoroughly: this will throw light on the whole of the material to be taught. Perhaps a clear indication of teaching method in the Federal Republic of Germany can be given by quoting an abridged extract from the directives for Nordrhein-Westphalia [8]:

Methodological

(*a*) Care, precision and clarity should be stressed.

(*b*) Mathematical aids and methods should be mastered by extensive practice.

1. Answers to a questionaire on Teaching Method are given in Appendix VIII.

(c) In the upper grades, the general solution is considered more important than numerical calculation.

(d) No stage of the solution should be omitted : these stages are :

1. clear conception of the problem.
2. hypothesis for possible solution.
3. thinking the hypothesis through.
4. choice of possible way to solution or group work for various ways to solution,
5. discussion. followed by an immaculate setting out in writing of the solution to the problem.

(e) Education of the critical faculties in the solution of problems: the value of estimation in avoiding errors. and limits of accuracy. are to be borne in mind.

Insight into mathematical relationships

The connection between number and space should be shown. The close relation between plane and space geometry should be shown. (This does not mean that they cannot be taught in separate courses).

Grasping of mathematical thinking

(a) The teaching should be concentrated largely on the elucidation of the essential modes of thinking. The methods and procedures should not appear to the pupil as unrelated ideas.

(b) Constant revision of fundamental knowledge is essential. It should not, however, be dealt with in a purely mechanical way, but against the background of rules, formulae and methods. There should be an opportunity for the pupil to display a fresh outlook on topics with which he is already familiar.

(c) The pupil should experience some pleasure in systematising and the pupil who is intellectually alert will question the acceptability of statements and declarations which he has mastered.

(d) From the beginning of his academic secondary education the pupil should learn to express mathematical facts accurately and in his own words. In the *Oberstufe* his understanding of mathematical methods and relationships must be demonstrated.

(e) On the whole, the method should be as follows:

1. from the particular to the general,
2. from the concrete to the abstract,
3. from individual cases to rules (induction).

Examples from applied mathematics will show the relationship of mathematics to the outside world.

Other points stressed by countries as regards methodology include:

(a) the need for clear, concise expression. 'The teacher can awaken the desire for precision by demanding exact words, definition and statements.'

(b) the teacher must collaborate with his colleagues in other disciplines so that the pupil is brought to see mathematics in relation to the other subjects he studies.

(c) the teacher must link his subject to reality. Thus, for example, the theory of possibility might be considered in relation to life insurance.

One country mentions that proposals have been made for the establishment of mathematical laboratories in schools where pupils could be taught to use calculating machines, computers etc. [1]

1. Answers to a questionnaire on changes in teaching method are given in Appendix X.

VIII. THE ACADEMIC SECONDARY LEAVING EXAMINATION IN MATHEMATICS

Most countries in the survey reserve the notion of a specialist education for the university and consider school the place for a general education. Potential university students must display a mastery of certain skills in order to be considered 'qualified for study', and the ability to think mathematically is one of these essential skills. Examples of other essential skills are: a mastery of the mother tongue and familiarity with its literature, a good knowledge of one foreign language. a fundamental knowledge of the natural sciences and a basic understanding of philosophical concepts. The terminal examination is usually, therefore, a certificate examination in which the student must obtain a good average in all the subjects which form part of the examination. If he obtains a good average he has manifested his mastery of the required skills and receives his certificate of academic maturity. Thus in most countries not only is the study of mathematics compulsory throughout the academic secondary school but it is usually one of the written papers in the terminal examination (even if the student is following a course with a non-scientific bias). [1] What is said by Norway may in fact characterise the attitude that mathematics is an integral part of any education:

" It is considered dangerous by many people to let academically trained students go out into the world without a minimum mastery of a subject which is so important in the society of today and will be even more so in that of tomorrow. " [1]

Sweden takes the viewpoint that mathematics is an aid in most subjects and occupations and gives this as the reason why all prospective students of the liberal arts must also study mathematics.

A notable exception to this attitude is the United Kingdom. The terminal examination is a subject examination and therefore. theoretically, any combination of subjects is possible. Usually, however a student's whole course of study is biased toward the 'Arts' or the 'Sciences' after the age of 16, as, for university entrance or further professional training there is an emphasis on special skills rather than a general education. In order to study a science subject at the university, for instance, a student requires good marks in that subject and one or two others (which will also be science subjects). One or two lessons a week may be devoted to English but a qualification in English is an official requirement by some universities only. The universities do ask for a more general spread of subjects at the lower level of the examination (taken usually at 16 years); specific requirements vary from one university to another but on the whole they ask for: a qualification in English language, English literature, a modern language and a science (not necessarily mathematics). This lower ordinary level of the examination is also a subject-type examination but the university usually specifies that a certain number of the subjects must have been taken at one sitting.

Mathematics is not a compulsory subject in the Certificate Examination in Ireland (Irish is the only compulsory subject, the other four being chosen by the student) but it is the only other subject besides Irish which is allotted the maximum of 600 marks, which gives an indication of its importance.

In the examination of mathematics, countries seem to divide into two groups: those whose system is partially centralised. as in the United Kingdom and France, in which the setting and the marking of the papers proceeds on an anonymous basis, but is in the main carried out by inspectors and teachers ; and those countries where the system is largely an internal one, with external surveillance, such as the Federal Republic of Germany and Belgium. In this latter group, however. inspectors and teachers are as before involved in both the setting of the question papers and the marking of the scripts.

1. An analysis of the secondary leaving examination in relation to mathematics is given in Appendix XI.

A special case is Sweden which is now moving over to a new system which will be in effective operation from 1968. The leaving examination as such has been abolished and in its stead teachers are using a method of continuous assessment. In the last two years of the gymnasium, teachers will carry out internally some 6 to 8 written examinations in mathematics. Results of these written examinations will be set against the observations by the teachers of the student's modes of work and habits of study and his response to oral questions throughout his schooling. In order to ensure comparability, the written tests given them will consist of standardised achievement tests, which are the responsibility of the National Board of Education The purpose of these achievement tests is to enable teachers to adjust their marks to the appropriate level by comparison with results of the tests themselves. At the same time, experts in mathematics and subject inspectors will go round the country advising teachers about the methods of evaluation.

All systems obviously have advantages and disadvantages. In partially centralised examinations, such as A Level in England or the *baccalauréat* in France, a comparatively greater objectivity *may* be obtained. On the other hand, this very objectivity carries with it the danger of divorcing the examinations from the teaching of the subject which, after all, should be the main concern of the teacher rather than preparation for the examination. The other system of internal (or decentralised) examinations, as in the Federal Republic of Germany, can lead obviously to a lack of comparability between the various schools or even the various *Länder*. But it does allow the mathematics examination to be linked much more closely to what the pupil has really learnt.

For those candidates following a mathematical bias in their course, a written examination is always demanded. On the other hand, as regards an oral examination, there would seem to be three distinct practices, which may be characterised as follows:

(1) No oral examination, as in the United Kingdom (but in the United Kingdom there is no oral examination in any subject except modern languages).

(2) An oral examination is possible, as in the Federal Republic of Germany. This may depend on the performance of the candidate in the examination, which must be in accordance with his work in his last year of study. If there is any discrepancy the examiners will demand an oral examination in order to ascertain the reasons.

(3) An oral examination is compulsory, as it is now once again in France. The length and form of this oral examination may vary considerably. In the Federal Republic of Germany, for example, a candidate may be given a subject to prepare which he may then study for half an hour and then be examined orally for some 15 to 20 minutes.

It would seem that most countries would prefer, if possible, to give an oral examination, but practical difficulties often militate against it.

In the written examination the time allocation would appear to differ widely in at least two respects. In the United Kingdom, for example, the candidate is given a number of questions to answer in a paper or papers that last usually 2½ hours. In the Federal Republic of Germany, by contrast, a candidate may be asked only one question and have 5 hours in which to answer it. Most countries insist upon only one examination paper, although the United Kingdom, because of its great specialisation, may set as many as four papers in pure and applied mathematics. In the Federal Republic of Germany, on the other hand, the one problem set is usually subdivided into a number of subsidiary problems which have to be solved.

The types of question set may be resolved into three different categories. There are, first, 'course questions' (Fr. *questions de cours):* these are questions which relate to a particular section of the course followed, which the good candidate should recognise with comparative ease. The second category is that of 'problems': these consist of questions in which a large amount of data is given and the candidate has to determine for himself what items of knowledge that he has learnt are necessary in order to solve them. The third category is that of the 'essay' (Ger. *Aufsatz*): this would seem to be a form of testing peculiar to the Federal Republic of Germany, in which the candidate is asked to write a short dissertation upon a particular problem, which is designed to test his capacity of mathematical thinking and of carrying out a process of logical argument. The Germans lay great stress on this particular ability. Most countries would, in fact, subscribe to the general prescription given by the Germans for mathematical questions in the *Abitur* when they say :

"Questions do not demand mere knowledge of fact but rather offer the candidate the opportunity of demonstrating his method of approaching a task, his intellectual maturity, his capacity for clear thinking and relevant self-expression... (and) are not so closely related to a task already studied in class as not to require fresh independent thinking." [21] p. 45.

IX. MATHEMATICS TEACHERS

The training of mathematics teachers

Some countries run concurrent programmes in the methodology of mathematics teaching with the basic mathematics course for the first degree. Ex mples of this are Austria, West Germany and Turkey; the more usual procedure, however, is for countries to have a separate period of study following upon the first degree course. There is, moreover, a wide variation between the amount of time devoted to what might be termed pedagogical study and practice te ching, as the examples given below show:

Practice teaching

Sweden 4-12 weeks
Norway 100 hours
Austria 1 year of internship teaching
Belgium, Federal Republic of Germany, Greece, Luxembourg 2 years' practice teaching under the supervision of expert teachers.

Education course

Sweden 4 months (practice and theory)
Norway 6 months' theory
Greece, Luxembourg 2 years' practice teaching and theory.

By contrast, Italy has no theory requirements or practice teaching period. In the United Kingdom, in addition to the university degree, there is an optional one-year theory course which includes a period of about 10 to 12 weeks' teaching practice.

The university course for mathematics teachers usually lasts at least 4 years and is concerned with the basic study of the subject matter. To this general pattern, however, there are notable exceptions: in Denmark the period of study lasts 5 to 6 years but intending mathematics teachers must also study physics and chemistry; in France the period may be as long as 6 years; in the Netherlands it may last a similar length of time; on the other hand, in the United Kingdom the normal degree course lasts only 3 years

Period of probation

Some countries insist upon newly qualified teachers undergoing a period of probation before they can become 'fully qualified', and appointments are not made permanent until this probationary period has been served. Others are satisfied with the combination of theoretical requirements and teaching practice after or during the initial university training (e.g. Norway). Where there is such a period of probation, the length of time during which the new teacher is under supervision varies considerably, from 3 months in the Netherlands to 6-8 years in Austria. In the United Kingdom probation lasts for 1 year but, as has been mentioned, the teacher training course is optional, so the future English academic secondary school teacher spends a maximum of 5 years in training (university course 3 years ; education course 1 year ; probation 1 year) ; whereas his German counterpart has a full total of 12 years (university course 5 years ; education course 2 years ; probation 5 years). Denmark seems to be unique in that, after the student has qualified and taught from 4 to 6 years he is required to return to the university for 2 years' further study, during which time his salary is paid.

The supply and demand of mathematics teachers

Most countries anticipate that the shortage of mathematics teachers will continue, possibly right up to 1970. Whilst the exact figures are difficult to ascertain, it would seem likely that, of the countries under review, only Greece and Luxembourg anticipate no shortfall in teacher supply. The projection of shortage of mathematics teachers in certain selected countries is shown in the table below. It should be noted that these figures are not entirely accurate because they also include science teachers and. in some cases, unspecialised teachers.

TABLE XII

Estimates of the shortfall in qualified full-time science and mathematics teachers at the secondary level in 1970

Country	Demand	Supply	Shortage
Denmark	800	522	278
England	23400	18200	5200
Netherlands	3075	2945	130
Norway	3100	2100	1000
Sweden	6350	6350	— [1]
Turkey	3043	1621	1412

1. It is likely, however, that there will be a shortage of *specialised* mathematics teachers.

(Source: *Supply, recruitment and training of science and mathematics teachers*, OECD, 1962) [22].

In view of the counter-attractions for mathematics graduates in other occupations, and the expanding demand for secondary education, it would seem unlikely that the shortage of fully qualified mathematics teachers can be filled for many years to come.

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APPENDICES

To: Section I

Appendix I : List of official publications giving aims, directives and programmes

Section III

Appendix II : Bloom's *Taxonomy of Educational Objectives*

Section V

Appendix III : Analysis of mathematics syllabuses

Appendix IV : Mathematics : analysis of topics specific to particular countries

Appendix V : Mathematics : time allocation

Appendix VI : Specimen timetables for academic secondary school showing time allocated to mathematics

A. Belgium : Catholic
B. Sweden
C. Luxembourg
D. Denmark
E. Netherlands

Section VI

Appendix VII : Projected general changes in the mathematics syllabus ; introduction of modern mathematics topics

Section VII

Appendix VIII : List of publications on teaching method

Appendix IX : The order in which the syllabus is taught

Appendix X : Changes in teaching method

Section VIII

Appendix XI : The nature of the secondary leaving examination in particular relation to mathematics

Appendix XII : List of topics and sub-topics prescribed for study in constituent states of the Council of Europe

APPENDIX 1

Summary of Questionaire asking mathematics experts to list Official Publications, giving aims, directives and programmes

AUSTRIA	*Verordnungsblatt für den Dienstbereich des Bundesministeriums für Unterricht (Österreichischer Bundesverlag)*, especially *Stück 10a*, 1955, Neuverlautbarung der Provisorischen Lehrpläne für die Mittelschulen.
BELGIUM	Ministère de l'Education Nationale: *Programme de mathématiques avec directions méthodologiques.*
CYPRUS	*Programmata Analytica Kai Orologia* (New Syllabuses following the reform of 1964) Programmata Analytica Kai Orologia didacteas Ylis National Printing Office Athens 1959 (top grades only).
FRANCE	Official syllabuses are published in the *Bulletin officiel de l'Education Nationale.* La Librairie Vuibert, 63, boulevard St-Germain, Paris-V^e^, publishes programmes of examinations. Official syllabuses also contain instructions by Inspector General — commenting on certain parts of syllabus and how they should be interpreted. The aims are not published officially but the comments mentioned above give some indications about these.
FEDERAL REPUBLIC OF GERMANY (Nordrhein-Westfalen)	*Richtlinien für den Unterricht in den Höheren Schulen. Mathematik.* Heft 8-10 der Schriftenreihe des Kultusministeriums. Henn-Verlag, Ratingen bei Düsseldorf. In this publication there are syllabuses, aims and directives as well as detailed recommendations on teaching method.
GREECE	These are mentioned in the official curriculum issued by the Ministry of Education.
IRELAND	Rules and Programmes for Secondary Schools.
ITALY	There are official Ministry syllabuses.
LUXEMBOURG	Ministry of National Education published each year *Horaires et Programmes.* Until 1962 the brochure contained methodological directives. In the last few years mathematics syllabuses have undergone and are undergoing notable modifications (modern mathematics). A complete reform of secondary education is before the Chamber at the moment.
NETHERLANDS	None: the information is contained in decrees *(Besluite)* issued as required.
NORWAY	Curriculum and Syllabuses are issued by the Ministry of Education.
SWITZERLAND	*Ordonnance de maturité fédérale.* There are also 22 *règlements cantonaux.*
SWEDEN	*Läroplan för gymnasiet* (Ed. by National Board of Education, Fack, Stockholm 22) contains curricula of all subjects for the new 'gymnasiet'.
TURKEY	The official Ministry of Education publication of 1957 is still in use.
UNITED KINGDOM	There are none for the 6th form. Programmes in the form of syllabuses for A level are published by each of the 8 examining boards.
FINLAND	Plans for the teaching of mathematics are approved by the Ministry of Education.

APPENDIX II

A Taxonomy of Educational Objectives

(Condensed from B. S. Bloom (ed). *Taxonomy of Educational Objectives -- The Classification of Educational Goals*, Handbook I: Cognitive Domain, Longmans, Green & Co. Ltd., London, 1956 (Reprinted 1964) [1]

1.00 *Knowledge*

Knowledge as defined here involves the recall of specifics and universals, the recall of methods and processes, or the recall of a pattern, structure or setting.

1.10 *Knowledge of specifics* — The recall of specific and isolable bits of information.

1.11 *Knowledge of terminology* — Knowledge of the referents for specific symbols (verbal and non-verbal), e. g. To define technical terms by giving their attributes, properties or relations.

1.12 *Knowledge of specific facts* — Knowledge of dates, events, persons, places etc., e. g. The possession of a minimum knowledge about the organisms studied in the laboratory.

1.20 *Knowledge of ways and means of dealing with specifics* — Knowledge of the ways of organizing, judging and criticizing.

1.21 *Knowledge of conventions* — Knowledge of characteristic ways of treating and presenting ideas and phenomena, e. g. Familiarity with the forms and conventions of the major types of works, such as scientific papers.

1.22 *Knowledge of trends and sequences* — Knowledge of the processes, directions and movements of phenomena with respect to time.

1.23 *Knowledge of classifications and categories* — Knowledge of the classes, sets, divisions, categories and arrangements which are regarded as fundamental for a given subject field.

1.24 *Knowledge of criteria* — Knowledge of the criteria by which facts, principles, opinions and conduct are tested or judged.

1.25 *Knowledge of methodology* — Knowledge of the methods of enquiry, techniques and procedures employed in a particular subject field. (Emphasis on knowledge of method rather than ability to use it.

1.30 *Knowledge of the universals and abstractions in a field* -- Knowledge of the major schemes and patterns by which phenomena and ideas are organized.

1.31 *Knowledge of principles and generalizations* -- Knowledge of particular abstractions which summarize observations of phenomena — abstractions of value in explaining, describing or predicting.

1.32 *Knowledge of theories and structures* -- Knowledge of the *body* of principles and generalizations together with their interrelations which present a clear and systematic view of a phenomenon, problem or field.

Intellectual Abilities and Skills

Abilities and skills refer to organized modes of operation and generalized techniques for dealing with materials and problems.

2.00 *Comprehension* — The lowest level of understanding: the individual knows what is being communicated and can make use of the material without necessarily being able to relate it to other material.

2.10 *Translation* — Comprehension as evidenced by the care and accuracy with which the communication is paraphrased or rendered from one language or form of communication to another.

1. Reprinted by kind permission of the Editor and David McKay Co. Inc., New York.

2.20 *Interpretation* — The explanation or summarization of a communication, involving a re-ordering, re-arrangement or new view of the material.

2.30 *Extrapolation* — The extension of trends or tendencies beyond the given data to determine implications, consequences, corollaries, effects etc.

3.00 *Application* — The use of abstractions in particular and concrete situations. Abstractions here may be general ideas, rules of procedure or generalized methods : or technical principles, ideas and theories which must be remembered or applied.

4.00 *Analysis* — The breakdown of a communication iuto its constituent elements or parts such that the relative hierarchy of ideas is made clear, or the relations between the ideas expressed are made explicit.

4.10 *Analysis of elements* — Identification of the elements included in a communication.

4.20 *Analysis of relationships* — The connections and interactions between elements and parts of a communicatiou.

4.30 *Analysis of organizational principles* — The organization, systematic arrangement and structure which hold the communication together — the explicit as well as the implicit structure.

5.00 *Synthesis* — The puttiug together of elements and parts so as to form a whole.

5.10 *Production of a unique communication* — The development of a communication in which a writer conveys ideas etc. to others.

5.20 *Production of a plan or proposed set of operations* — The development of a plan of work or the proposal of a plan of operations e.g. ability to propose ways of testing hypotheses.

5.30 *Derivation of a set of abstract relations* — The development of a set of abstract relations either to classify or explain particular data or the deduction of propositions and relations from a set of basic propositions or symbolic representations.

6.00 *Evaluation* — Judgements about the value of material and methods for given purposes.

6.10 *Judgement in terms of internal evidence* — Evaluation of accuracy of a communication by judging logical accuracy, consistency etc.

6.20 *Judgement in terms of external criteria* — Evaluation of material with reference to selected or remembered criteria.

APPENDIX III[1]

Mathematics Syllabus

ANALYSIS

Topic		Au	Bg	Cy	Fr	FRG	Gr	Lu	Ir	It	Ne	No	Sd	Sw	Tu	UK	Fi
History of Maths																	
	509.	x		x		x	x						x		x		
Numeration Systems	511.10		x	x	x	x	x		x		x		x		x		x
	.11			x		x	x		x				x				
Algebra	512.02.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	.03	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	.04	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
	.05		x	x	x	x	x	x	x	x	x	x	x	x	x	x	
	.06		x	x	x		x	x				x		x	x	x	
	.07	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	.08		x	x	x	x	x	x	x	x	x	x	x		x	x	x
Imaginary quantities	.2																
	.21	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	.241	x	x	x	x		x	x	x	x			x	x	x	x	x
	.242	x	x	x	x		x	x	x				x	x		x	
	.243	x	x	x	x			x	x					x	x	x	x
Combinatory Analysis	.5	x	x	x	x		x	x	x	x			x	x		x	
Abstract algebra	.800			x	x		x	x	x				x	x			
	.801			x	x		x	x	x				x	x			
	.802			x	x		x	x	x				x	x			
	.803			x	x		x	x	x				x	x			
	.804			x	x		x	x	x				x	x			
	.805						x										
	.806			x	x		x	x	x				x	x			
	.807			x	x		x		x								
	.808			x	x		x		x					x			
	.809				x			x					x	x			
Theory of numbers	.810	x	x	x	x	x	x		x		x	x	x	x	x		x
	.8101			x			x					x					
	.8102	x			x				x	x	x	x	x		x		x
	.8103	x			x				x	x	x	x	x		x		x
	.8104	x		x	x		x	x	x	x	x	x	x	x			
	.8105			x			x			x		x		x			
	.8106	x		x	x		x	x	x	x			x	x	x		
	.8121	x		x	x	x	x	x		x		x		x	x		
	.8122													x			
	.815			x			x										
	.8151			x	x		x	x						x			
Theory of Equations	.821	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
	.822	x	x	x	x		x	x		x	x	x		x		x	
Determinants	.831		x	x			x	x		x				x		x	
	.832		x	x			x									x	
	.833		x														
Group Theory	.861			x	x		x	x						x			
	.862							x									

1. A key to the sub-topics, each here represented by a decimal fraction, is given in Appendix XII.

Topic		Au	Bg	Cy	Fr	FRG	Gr	Lu	Ir	It	Ne	No	Sd	Sw	Tu	UK	Fi
Vector algebra	512.8951		x	x	x	x	x	x	x		x	x	x	x	x	x	x
	.8952			x	x	x	x	x	x		x		x	x	x	x	x
	.8953			x	x		x	x	x				x	x		x	
	.8954			x			x	x	x					x		x	
	.8955													x			
	.8956			x			x						x				
	.8957																
	.8958			x			x										
	.896							x						x			
	.8961																
	.8962													x			
Elementary plane geometry	513.11	x	x	x	x		x		x	x	x				x	x	x
	.12		x	x	x		x		x						x		
Solid geometry	.31	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Modern geometry	.51		x		x			x	x	x					x	x	
	.52	x	x		x			x			x			x	x		x
	.53		x		x				x						x		
	.54		x		x			x							x		
	.55		x												x		
	.56		x											x			x
	.57																
	.58		x													x	
	.59		x														x
	.591		x														
Plane trigonometry	514.51	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x
Spherical geometry	.61	x	x											x			
	.62	x	x											x			
	.63	x															
	.64				x												
	.65	x												x			
	.66																
	.67																
Descriptive geometry	515	x	x			x		x			x		x	x			
	.4	x			x												
Straight lines	516.11	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x
Curves in Cartesian form	.21	x	x	x	x		x	x	x	x	x	x		x	x	x	x
	.22	x	x	x	x		x	x							x	x	x
	.23	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
	.24	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
Lines and plane in Cartesian form	.31				x								x	x		x	
	.32				x			x					x	x		x	
Modern algebraic geometry	.501	x	x					x	x				x	x		x	
	.502		x						x					x			
	.503		x					x	x	x				x	x	x	
	.504		x														
	.505								x							x	
	.506		x		x				x						x		
	.507	x	x					x	x					x	x	x	
	.508		x						x						x	x	
Geometrical transformations	.551	x	x		x			x	x		x				x	x	
	.552		x					x						x	x		
	.56								x						x		
Differential geometry	.57										x				x	x	

Topic		Au	Bg	Cy	Fr	FRG	Gr	Lu	Ir	It	Ne	No	Sd	Sw	Tu	UK	Fi
Kinematic geometry	516.81				x	x			x				x		x	x	x
	.82				x	x									x	x	
	.83				x	x									x	x	
	.84				x												
Infinitesimal calculus	517.11	x	x	x	x		x		x	x	x		x	x	x	x	
	.12	x	x	x			x	x	x		x		x	x		x	x
	.13		x	x				x	x					x			x
	.14	x	x	x	x		x	x		x	x		x	x	x	x	x
	.15	x	x	x	x		x	x	x	x	x		x	x	x		
	.16			x			x										
	.17			x			x		x							x	
Differential calculus	.200	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
	.20.		x					x						x			
Series	.210						x	x					x	x		x	
	.211							x	x					x		x	
	.212												x	x			
	.213							x					x	x		x	
	.214			x			x	x								x	
	.215	x	x	x	x	x	x	x	x			x	x	x	x	x	x
	.216	x		x			x		x				x	x	x	x	x
	.217		x	x	x		x	x	x	x		x	x	x	x	x	x
	.218		x	x	x		x	x	x			x	x	x			x
Variation of function values	.271		x					x		x				x			
	.272		x		x					x			x	x			
	.273	x	x	x	x		x	x		x	x		x	x	x	x	x
	.274		x														
Differential calculus problems	.291	x														x	
	.292																
	.293											x			x	x	
Integral calculus	.30	x		x	x		x	x	x	x	x	x	x	x	x	x	
	.31						x									x	
Integration	.312	x					x	x	x	x		x	x	x	x	x	
	"	x			x		x	x				x	x	x	x	x	
	"						x	x		x		x		x	x	x	
	"						x		x			x	x	x	x	x	
	"						x					x				x	
Integration as inverse of diff.	517.313	x			x		x	x	x	x		x	x	x	x	x	x
Differential equations	.38				x				x				x		x	x	
	"												x			x	
	"				x				x				x			x	
	"												x			x	
	"														x	x	
	"												x			x	
Integral calculus problems and tables	.390	x			x			x	x	x	x	x	x	x	x	x	
	.391	x			x			x	x	x	x	x	x	x	x	x	
	.392															x	
	.393																
	.394													x	x	x	
	.395								x					x	x	x	x
Calculus of functions	.521	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x
	.522	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x
	.523						x									x	
	.524		x	x			x	x	x				x	x		x	x
	.525															x	

Topic		Country															
		Au	Bg	Cy	Fr	FRG	Gr	Lu	Ir	It	Ne	No	Sd	Sw	Tu	UK	Fi
Calculus of finite differences	517.61/2								x					x		x	
	.63												x			x	
	.64												x			x	
	.81																
Probabilities	519.1	x		x			x	x	x	x			x	x		x	
Treatment of data	.8			x			x	x	x				x			x	
Statistical maths	.900			x					x				x			x	
	.901								x				x			x	
	.902								x				x			x	
	.903												x			x	
	.904								x				x			x	
	.905												x			x	
	.906												x			x	
	.907															x	
	.908															x	
	.909															x	
	.910															x	
	.911															x	
	.912															x	
	.913															x	
	.92								x				x				
Kinematics	531.1				x	x						x			x	x	x
	”					x									x	x	
	”					x						x			x	x	
	”					x									x	x	x
	”					x									x	x	x
	”																
SHM's	.32					x						x			x	x	
	”				x	x						x			x	x	
	”				x										x	x	
																x	

APPENDIX IV

Mathematics

(Analysis of topics specific to particular countries)

TOPICS SPECIFIC TO AUSTRIA

509	History of mathematics
512.241	Operations with complex numbers
.242	De Moivre's theorem
.243	Solution of trig. equations involving all relevant angles
.5	Combinatory analysis
.810	Natural numbers
.8102	Extension to integers
.8103	Extension to rationals
.8104	Properties of real numbers
.8106	Complex numbers
.812	Divisibility
.8121	Euclid's algorithm (not polynomials)
.821	Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
513.11	Triangle and associated circles
.52	Similitude and homothety
514.61	Right angles spherical triangle
.62	Spherical triangles in general
.63	Geo-sphere: great circle navigation
.65	Celestial sphere; hour angle; R.A.; declination etc.
.66	Equation of time
515	*Descriptive Geometry*
	Stereometry; use of graphical and semi-graphical methods; descriptive geometry — solve problems of distance and angle
.4	Real conical projection
516.22	Orthogonal projection — ellipse as projection of circle
.31	Distances and angles between planes; 2 lines; line and plane
.501	Polar co-ordinates
.507	General equation of 2nd degree in two variables
.551	Pole and polar
517.11	lim F (n) $\rightarrow \infty$
.12	Series; convergence
.14	$\lim_{n \rightarrow a} F(n)$
.15	Continuity
.215	Arithmetic geometric series
.216	Summation of 'finite series'; induction
.273	Maxima, minima, inflexions
.291	Curvature
.30	Definition and properties of the definite integral
.312	Techniques of integration Substitution
.313	Concept of integrating as the inverse form of differentiating
.390	Areas assuming existence
.391	Volume of solids of revolution
.521	Logarithms and exponential functions
.522	Circular functions
519.1	Probabilities

TOPICS SPECIFIC TO BELGIUM

511.10	Notational systems
512.05	Identities

512.06 Partial fractions
.08 Inequalities (involving modification. squaring. square root; 2nd degree)
.241 Operations with complex numbers
.242 De Moivre's theorem
.243 Solution of trig. equations involving all relevant angles
.5 Combinatory analysis
.8151 Properties of a field, ring
.821 Simultaneous eq··tions (3 linears, 1 linear, 1 2nd degre·)
.822 Conditions for ns' t ·y up to 3 linear simultaneous equ·tions
.831 Determinants up t ›rder
.832 Higher order
.833 Product of determinants
.8951 Addition of vectors
.8952 Multiplication by a scalar
513.11 Triangle and associated circles ; Simson line ; Circle of Apollonius
.12 Ceva and Menelaus's theorems
.51 Coaxial circles and associated systems
.52 Similitude and homothety
.53 Inversion in plane
.54 Reciprocation of w.r. to circle
.55 Duality: Pascal and Brianchon's theorems
56 Homogeneous co-ordinates (*a*) Cartesian. (*b*) Triangular, (*c*) Ger..al
.58 Properties of line pairs
.59 Involution systems on a straight line
.591 General homographies — line at infinity. circular points
514.61 Right angles spherical triangles
.62 Spherical triangles in general
515 *Descriptive Geometry*
Stereometry. Use of graphical and semi-graphical methods; description geometry -- solve problems and angles
516.22 Orthogonal projection — ellipse as projection of circle;
.501 Polar co-ordinates
.502 Graphs of well known curves, cycloids, epicycloids and hypocycloids, cardiods
.503 Curve tracing; asymptotes; double points, curves inflexions etc. both from Cartesian equations, polar equations: co-ordinates in terms of a parameter
.504 Pedal (p,r) equations φ; equations; equations of the pedal etc.
.505 Cross ratio
.506 Quadrilateral and quadrangular
.507 General equation of 2nd degree in two variables
.508 Family $S + kS' = 0$
.551 Pole and polar
.552 Invariants associated with translations, ·otations, reflexions. enlargements, sheers etc. Products of transformations
.56 Harmonic ranges and pencils treated parametrically
517.11 $\lim F(n)\ n \to \infty$
.12 Series; convergence
.13 Tests for convergence
.14 $\lim_{n \to a} F(n)$
.15 Continuity
.201 Partial differentiation
.215 Arithmetic eometric series
.216 Summation of 'finite series'; induction
.217 Binomial theorems for positive integral exponent
.218 Relation between co-efficients
.271 Rolle's theorem for polynomial functions, with proof
.272 Rolle's theorem in general (intuitive approach but precise statement)
.273 Maxima, minima, inflexions
.274 Order of contact
.521 Logarithms and exponential functions
.522 Circular functions
.524 Inverse circular functions

TOPICS SPECIFIC TO CYPRUS

509	History of mathematics
511.10	Notational systems
.11	Binary arithmetic (application to computers)
512.05	Identities
.06	Partial fractions
.08	Inequalities (involving modification, squaring, square root: 2nd degree)
.241	Operations with complex numbers
.242	De Moivre's theorem
.243	Solution of trig. equations involving all relevant angles
.5	Combinatory analysis
.800	Set; subset
.801	Inclusion, intersection, union
.802	Empty set
.803	Complementation
.804	Equivalence relations; order relations
.806	Mapping, function
.807	Many — one
.808	One — one
.810	Natural numbers
.8101	Peano's axioms
.8104	Properties of real numbers
.8105	Dedekind section (or any alternative construction of real numbers)
.8106	Complex numbers
.812	Divisibility
.8121	Euclid's algorithm (not polynomials)
.815	Algebraic fields
.8151	Properties of a field, ring
.821	Simultaneous equations (3 linears, 1 linear. 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
.831	Determinants up to 3rd order
.832	Higher order
.861	Group
.8951	Addition of vectors
.8952	Multiplication by a scalar
.8953	Scalar product
.8954	Vector product
.8956	Differentiation
.8958	Operator Δ
513.11	Triangle and associated circles; Simson line; Circle of Apollonius
.12	Ceva and Menelaus's theorems
516.22	Orthogonal projection — ellipse as projection of circle
517.11	lim F (n) $n \to \infty$
.12	Series; convergence
.13	Tests for convergence
.14	$\lim_{n \to a} F(n)$
.1:	Continuity
.16	Uniform convergence
.17	Definite integral. Convergence
.214	Arc tan x
.215	Arithmetic geometric series
.216	Summation of 'finite series'; induction
.217	Binomial theorems for positive integral exponent
.218	Relation between coefficients
.273	Maxima, minima, inflexions
.30	Definition and properties of the definite integral
.52	Functions of real variables
.521	Logarithms and exponential functions
.522	Circular functions
.524	Inverse circular functions
519.1	Fundamental laws of probability

519.8 Presentation ; pie charts, histograms, frequency. polygons
.900 Mean, mode, median, deciles, percentiles

TOPICS SPECIFIC TO FRANCE

511.10 Notational systems
512 05 Identities
.06 Partial fractions
.08 Inequalities (involving modification, squaring, square root: 2nd degree)
.214 Operations with complex numbers
.242 De Moivre's theorem
.243 Solution for trig. equations involving all relevant angles
.5 Combinatory analysis
.800 Set; subset
.801 Inclusion, intersection
.802 Empty set
.803 Complementation
.804 Equivalence relations; order relations
.806 Mapping. function
.807 Many — one
.808 One — one
.810 Natural numbers
.8102 Extension to integers
.8103 Extension to rationals
.8104 Properties of real numbers
.8106 Complex numbers
.812 Divisibility
.8121 Euclid's algorithm (not polynomials)
.815 Algebraic fields
.8151 Properties of a field, ring
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822 Conditions for consistency of up to 3 linear simultaneous equations
.861 Group
.8951 Addition of vectors
.8952 Multiplication by a scalar
.8953 Scalar product
513.11 Triangle and associated circles; Simson line; circle of Apollonius
.12 Ceva and Menelaus's theorems
.51 Coaxial circles and associated systems
.53 Inversion in plane
.54 Reciprocation of w.r. to circle
514.64 Map projection
.65 Celestial sphere; hour angle: R.A.; declination etc.
515.4 Real conical projections
516.22 Orthogonal projection — ellipse as projection of circle
.31 Distances and angles between planes; two lines; line and plane
.32 3 dimensional analytical geometry of plane and straight line
.506 Quadrilateral and quadrangular
.81 Position vector; velocity vectors, acceleration vector
.82 Uniform acceleration in a straight line
.83 Uniform acceleration in a plane; parabolic motion
.84 Hodograph
517.11 lim F (n) $n \to \infty$
.14 lim F (n) $n \to a$
.15 Continuity
.215 Arithmetic and geometric series
.217 Binomial theorems for positive integral exponent
.218 Relation between coefficients
.272 Rolle's theorem in general (intuitive approach but precise statement)
.273 Maxima, minima, inflexions
.312 Techniques of integration. Partial fraction
.313 Concept of integrating as the inverse of differentiating

.38 Differential equations

$$\frac{dy}{dx} = ky$$

$$\frac{d^2y}{dx^2} + n^2y = 0$$

.390 Areas assuming existence
.391 Volumes of solids of revolution
.521 Logarithms and exponential functions
.522 Circular functions
531.1 *Kinematics*
Motion in a circle with uniform speed
531.32 *Simple harmonic motions*
Simple sine wave motion

TOPICS SPECIFIC TO THE FEDERAL REPUBLIC OF GERMANY

509 History of mathematics
511.10 Notational systems
.11 Binary arithmetic (application to computers)
512.05 Identities
.08 Inequalities (involving modification. squaring, square root: 2nd degree)
.810 Natural numbers
.8104 Properties of real numbers
.8105 Dedekind section (or any alternative construction of real numbers)
.812 Divisibility
.8121 Euclid's algorithm (not polynomials)
.8951 Addition of vectors
.8952 Multiplication by a scalar
515 *Descriptive Geometry*
Stereometry. Use of graphical and semi-graphical methods; descriptive geometry — solve problems of distance and angle
516.22 Orthogonal projection — ellipse as projection of circle
.81 Position vector; velocity vectors; acceleration vector
.82 Uniform acceleration in a straight line
.83 Uniform acceleration in a plane; parabolic motion
517.11 lim F (n) $n \to \infty$
.12 Series; convergence
.13 Tests for convergence
.14 lim F (n) $n \to a$
.15 Continuity
.17 Infinite integral. Convergence
.216 Summation of finite series; induction
.273 Maxima, minima, inflexions
.291 Curvature
.293 Leibniz's theorem on differentiation of product
.30 Definition and properties of the definite integral
.313 Concept of integrating as the inverse of differentiating
.390 Areas assuming existence
.391 Volumes of solids of revolution
.521 Logarithms and exponential functions
531.1 *Kinematics*
Motion in a circle with uniform speed
Motion of a particle (heavy) constrained to move in a vertical circle
Radial and transverse components of acceleration
Tangential and normal components
Motion in a plane with acceleration towards a fixed point
.32 *Simple harmonic motions*
Simple sine wave motion

TOPICS SPECIFIC TO GREECE

509 History of mathematics
511.10 Notational systems

511.11	Binary arithmetic (application to computers)
512.05	Identities
.06	Partial fractions
.08	Inequalities (involving modification, squaring, square root: 2nd degree)
.241	Operations with complex numbers
.242	De Moivre's theorem
.243	Solution of trig. equations involving all relevant angles
.5	Combinatory analysis
.800	Set; subset
.801	Inclusion. intersection, union
.802	Empty set
.803	Complementation
.804	Equivalence; order relations
.805	Boolean algebra
.806	Mapping function
.807	Many — one
.808	One — one
.810	Natural numbers
.8101	Peano's axioms
.8104	Properties of real numbers
.8105	Dedekind Section (or any alternative construction of real numbers)
.8106	Complex numbers
.812	Divisibility
.8121	Euclid's algorithm (not polynomials)
.815	Algebraic fields
.8151	Properties of a field, ring
.821	Simultaneous equations (3 linears. 1 linear, 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
.831	Determinants up to 3rd order
.832	Higher order
.861	Group
.8951	Addition of vectors
.8952	Multiplication by a scalar
.8953	Scalar product
.8954	Vector product
.8956	Differentiation
.8958	Operator Δ
513.11	Triangle and associated circles; Simson line; circle of Apollonius
.12	Ceva and Menelaus's theorems
516.22	Orthogonal projection — ellipse as projection of circle
517.11	lim F (n) $n \rightarrow \infty$
.12	Series; convergence
.13	Tests for convergence
.14	lim F (n) $n \rightarrow a$
.15	Continuity
.16	Uniform convergence
.17	Infinite integral. Convergence
.21	Series
.214	Arc tan x
.215	Arithmetic geometric series
.216	Summation of 'finite series'; induction
.217	Binomial theorems for positive integral exponent
.218	Relation between coefficients
.273	Maxima, minima, inflexions
.30	Definition and properties of the definite integral
.312	Techniques of integration Substitution Partial fractions Parts Reduction formulae
.313	Concept of integrating as the inverse of differentiating
.39	Integral calculus problems and tables
.521	Logarithms and exponential functions

517.522 Circular functions
.523 Hyperbolic functions
.524 Inverse circular functions
519.1 Fundamental laws of probability
.8 Presentation; pie charts, histograms, frequency, polygons

TOPICS SPECIFIC TO IRELAND

511.10 Notational systems
.11 Binary arithmetic (application to computers)
512.05 Identities
.08 Inequalities (involving modification, squaring, square root: 2nd degree)
.241 Operations with complex numbers
.242 De Moivre's theorem
.243 Solution of trig. equations involving all relevant angles
.5 Combinatory analysis
.800 Set, subset
.801 Inclusion, intersection, union
.802 Empty set
.803 Complementation
.804 Equivalence relations; order relations
.806 Mapping, function
.807 Many — one
.808 One — one
.810 Natural numbers
.8102 Extension to integers
.8103 Extension to rationals
.8104 Properties of real numbers
.8106 Complex numbers
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.8951 Addition of vectors
.8952 Multiplication by a scalar
.8953 Scalar product
.8954 Vector product
513.1 Elementary plane geometry
.12 Ceva and Menelaus's theorems
.51 Coaxial circles and associated systems
.53 Inversion in plane
.54 Reciprocation of w.r. to circle
516.501 Polar co-ordinates
.502 Graphs of well known curves, cycloids, epicycloids and hypocycloids, cardiods
.503 Curve tracing; asymptotes: double points, curves inflexions etc. both from Cartesian equations, polar equations: co-ordinates in terms of a parameter
.505 Cross ratio
.506 Quadrilateral or quadrangular
.507 General equation of 2nd degree in two variables
.508 Family $S + kS' = 0$
.551 Pole and polar
.56 Harmonic ranges and pencils treated parametrically
.81 Position vector; velocity vectors; acceleration vector
.82 Uniform acceleration in a straight line
.83 Uniform acceleration in a plane; parabolic motion
.84 Hodograph
517.11 $\lim F(n)\ n \to \infty$
.12 Series; convergence
.13 Tests for convergence
.14 $\lim F(n);\ n \to a$
.15 Continuity
.17 Infinite integral. Convergence
.211 Binomial series, logarithms, exponential series
.215 Arithmetic geometric series
.216 Summation of 'finite series'; induction
.217 Binomial theorems for positive integral exponent
.218 Relation between coefficients

517.30	Definition and properties of the definite integral
.312	Techniques of integration Substitution Reduction formulae
.313	Concept of integrating as the inverse of differentiation
.38	Differential equations $\frac{dy}{dx} = ky$ $\frac{d^2y}{dx^2} + n^2y = 0$
.390	Areas assuming existence
.391	Volumes of solids of revolution
.395	Areas of surface of revolution
.521	Logarithms and exponential functions
.522	Circular functions
.524	Inverse circular functions
.61	Numerical methods
.62	Solution of numerical equations, Newton, Morner etc.
519.1	Fundamental laws of probability
.8	Presentation; pie charts, histograms, freqnency, polygons
.900	Mean, mode, median, deciles, percentiles
.901	Measures of dispersion. variance
.902	Binomial distribution
.904	Normal — Gaussian — distribution
.92	*Programming* Simple linear programmes
531.1	*Kinematics* Motion in a circle with uniform speed, motion of a particle (heavy) constrained to move in a vertical circle, radial and transverse components of acceleration, tangential and normal components, motion in a plane with acceleration towards a fixed point, orbits
.32	*Simple harmonic motions* Simple sine wave motion Combinations of SHMs in a plane — equal frequencies

TOPICS SPECIFIC TO ITALY

512.05	Identities
.08	Inequalities (involving modification. squaring, square root: 2nd degree)
.241	Operations with complex numbers
.5	Combinatory analysis
.8102	Extension to integers
.8103	Extension to rationals
.8104	Properties of real numbers
.8105	Dedekind section (or any alternative construction of real numbers)
.8106	Complex numbers
.812	Divisibility
.8121	Euclid's algorithm (not polynomials)
.8151	Properties of a field, ring
.821	Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
.831	Determinants up to 3rd order
513.1	Elementary plane geometry
.11	Triangle and associated circles
516.503	Curve tracing; asymptotes; double points, curves inflexions etc. both from Cartesian equations, polar equations: co-ordinates in terms of a parameter
.7	Differential geometry
.81	Position vector; velocity vectors; acceleration vector
.82	Uniform acceleration in a straight line
.83	Uniform acceleration in a plane; parabolic motion
.84	Hodograph
517.11	$\lim F(n)\ n \to \infty$
.14	$\lim F(n)\ n \to a$
.15	Continuity
.217	Binomial theorems for positive integral exponent
.271	Rolle's theorem for polynomial functions, with proof

517.272	Rolle's theorem in general (intuitive approach but precise statement)
.273'	Maxima, minima, inflexions
.30	Definition and properties of the definite integral
.312	Techniques of integration
	Substitution
	Parts
	Reduction formulae
.313	Concept of integrating as the inverse of differentiating
.39	Integral calculus problems and tables
.390	Areas assuming existence
.391	Volumes of solids of revolution
.521	Logarithms and exponential functions
.522	Circular functions
519.1	Fundamental laws of probability
531.1	*Kinematics*
	Motion in a circle with uniform speed
	Motion of a particle (heavy) constrained in a vertical circle
	Radial and transverse components of acceleration
	Tangential and normal components
	Motion in a plane with acceleration towards a fixed point
	Orbits

TOPICS SPECIFIC TO LUXEMBOURG

512.05	Identities
.06	Partial fractions
.08	Inequalities (involving modification, squaring, square root: 2nd degree)
.241	Operations with complex numbers
.242	De Moivre's theorem
.243	Solution of trig. equations involving all relevant angles
.5	Combinatory analysis
.800	Set; subset
.801	Inclusion, intersection, union
.802	Empty set
.803	Complementation
.804	Equivalence relations; order relations
.806	Mapping, function
.808	One — one
.809	Vector space
	Neighbourhood
	Topology and topological spaces
.810	Natural numbers
.8104	Properties of real numbers
.8106	Complex numbers
.8121	Euclid's algorithm (not polynomials)
.8151	Properties of a field, ring
.821	Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
.831	Determinants up to 3rd order
.861	Group
.862	Euclidean geometry as study of invariants under group transformation
.8951	Addition of vectors
.8952	Multiplication by a scalar
.8953	Scalar product
.8954	Vector product
.896	Matrices 2×2; 3×3
513.51	Coaxial circles and associated systems
.52	Similitude and homothety
515	Descriptive geometry — solve problem of distance and angles
516.22	Orthogonal projection — ellipse as projection of circle
.32	3 dimensional analytic geometry of plane and straight line
.501	Polar co-ordinates
.503	Curve tracing; double points, curves inflexions, etc., both for Cartesian equations, polar equations: co-ordinates in terms of a parameter

516.507 General equation of 2nd degree in two variables
.551 Pole and polar
.552 Invariants associated with translations, rotations, reflexions, enlargements, sheers etc. — products of transformation
517.12 Series; convergence
.13 Test for convergence
.14 lim F (n) n → a
.15 Continuity
.201 Partial differentiation
.210 Use of Taylor's and Maclaurin's theorems - - assuming their existence
.211 Binomial series, logarithms, exponential series
.213 Series for sin x. cos x
.214 Arc tan x
.215 Arithmetic geometric series
.217 Binomial theorems for positive integral exponent
.218 Relation between coefficients
.271 Rolle's theorem for polynomial functions, with proof
.273 Maxima, minima, inflexions
.30 Definition and properties of the definite integral
.312 Techniques of integration
Substitution
Partial fraction
Parts
.313 Concept of integrating as the inverse of differentiating
.390 Areas assuming their existence
.391 Volumes of solids of revolution
.521 Logarithms and exponential functions
.522 Circular functions
.524 Inverse circular functions
519.1 Fundamental laws of probability
.8 Functions of complex variables

TOPICS SPECIFIC TO THE NETHERLANDS

511.10 Notational systems
512.05 Identities
.08 Inequalities (involving modification, squaring, square root: 2nd degree)
.810 Natural numbers
.8102 Extension to integers
.8103 Extension to rationals
.8104 Properties of real numbers
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822 Conditions for consistency of up to 3 linear simultaneous equations
.8951 Addition of vectors
.8952 Multiplication by a scalar
513.1 Elementary plane geometry
.11 Triangle and associated circles
.52 Similitude and homothety
515 *Descriptive geometry*
Stereometry. Use of graphical and semi-graphical methods; description geometry — solve problems of distance and angles
516.551 Pole and polar
517.11 lim F (n) n → ∞
.12 Series ; convergence
.14 lim F (n) n → a
.15 Continuity
.273 Maxima, minima, inflexions
.30 Definition and properties of the definite integral
.390 Areas assuming existence
.391 Volumes of solids of revolution
.521 Logarithms and exponential functions
.522 Circular functions

TOPICS SPECIFIC TO NORWAY

512.05	Identities
.06	Partial fractions
.08	Inequalities (involving modification. squaring. square root : 2nd degree)
.241	Operations with complex numbers
.242	De Moivre's theorem
.243	Solution of trig. equations involving all relevant angles
.5	Combinatory analysis
.810	Natural numbers
.8101	Peano's axioms
.8102	Extension to integers
.8103	Extension to rationals
.8104	Properties of real numbers
.8105	Dedekind section (or any alternative construction of real numbers)
.812	Divisibility
.8121	Euclid's algorithm (not polynomials)
.821	Simultaneous equations (3 linears, 1 linear. 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
.831	Determinants of up to 3rd order
.8951	Addition of vectors
513.1	Elementary plane geometry
.11	Triangle and associated circles
516.7	Differential geometry
517	Calculus
.21	Series
.215	Arithmetic geometric series
.217	Binomial theorems for positive integral exponent
.218	Relation between coefficients
.271	Rolle's theorem for polynomial functions. with proof
.273	Maxima, minima, inflexions
.293	Leibnitz's theorem on differentiation of product
.30	Definition and properties of the definite integral
.312	Techniques of integration
	Substitution
.313	Concept of integrating as the inverse of differentiating
.39	Integral calculus problems and tables
.390	Areas assuming existence
.391	Volumes of solids of revolution
.6	Calculus of finite differences
531.1	*Kinematics :* Motion in a circle with uniform speed ; tangential and normal components
.32	Simple harmonic motion
	Simple sine wave motion

TOPICS SPECIFIC TO SWEDEN (New *Gymnasiet*)

511.10	Notational systems
.11	Binary arithmetic (application to computers)
512.05	Identities
.08	Inequalities (involving modification, squaring, square root : 2nd degree)
.241	Operations with complex numbers
.242	De Moivre's theorem
.5	Combinatory analysis
.800	Set ; subset
.801	Inclusion, intersection, union
.802	Empty set
.803	Complementation
.804	Equivalence relations; order relations
.806	Mapping, function
.809	Vector space
	Neighbourhood
.8106	Complex numbers
.821	Simultaneous equations (3 linears, 1 linear, 1 2nd degree)

512.8951	Addition of vectors
.8952	Multiplication by a scalar
.8953	Scalar product
.8956	Differentiation
515	*Descriptive Geometry*
	Stereometry. Use of graphical and semi-graphical methods ; description geometry — solve problems of distance and angles
516.31	Distance and angles between planes ; 2 lines ; line and plane
.32	3 dimensional analytic geometry of plane and straight line
.81	Position vector ; velocity vectors ; acceleration vector
517.11	lim F (n) $n \to \infty$
.12	Series ; convergence
.14	lim F (n) $n \to a$
.15	Continuity
.210	Use of Taylor's and Maclaurin's theorems — assuming their existence
.212	Existence of Taylor's series and the remainder after n terms
.213	Series for sin x, cos x
.215	Arithmetic geometric series
.216	Summation of 'finite series' induction
.217	Binomial theorems for positive integral exponent
.218	Relation between coefficients
.272	Rolle's theorem in general (intuitive approach but precise statement)
.30	Definition and properties of the definite integral
.312	Techniques of integration
	Substitution
	Parts
.313	Concept of integrating as the inverse of differentiating
.38	Differential equations

$$\frac{dy}{dx} = ky$$

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c = 0$$

$$\frac{d^2y}{dx^2} + n^2y = 0$$

$$\frac{dy}{dx} + Py = Q$$

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

.390	Areas assuming existence
.391	Volumes of solids of revolution
.521	Logarithms and exponential functions
.522	Circular functions
.534	Inverse circular functions
.63	Evalution of definite integrals. Simpson, Gauss etc.
.64	Approx to functions by polynomials
519.1	Fundamental laws of probability
.8	Presentation ; pie charts, histograms, frequency, polygons
.900	Mean, mode, median, deciles, percentiles
.901	Measures of dispersion, variance
.902	Binomial distribution
.903	Poisson distribution
.904	Normal-Gaussian-distribution
.905	Sampling : estimation of population parameters
.906	Tests for significance for small samples
.92	Simple linear programmes

TOPICS SPECIFIC TO SWITZERLAND

512.05	Identities
.06	Partial fractions

512.241	Operations with complex numbers
.242	De Moivre's theorem
.243	Solution of trig. equations involving all relevant angles
.5	Combinatory analysis
.800	Set ; subset
.801	Inclusion, intersection, union
.802	Empty set
.803	Complementation
.804	Equivalence relations ; order relations
.806	Mapping, function
.807	Many — one
.808	One — one
.809	Vector space
	Neighbourhood
	Topology and topological spaces
.810	Natural numbers
.8104	Properties of real numbers
.8105	Dedekind section (or any alternative construction of real numbers)
.8106	Complex numbers
.812	Divisibility
.8121	Euclid's algorithm (not polynomials)
.8122	Congruences
.815	Algebraic fields
.8151	Properties of a field. ring
.821	Simultaneous equations (3 linears. 1 linear, 1 2nd degree)
.822	Conditions for consistency of up to 3 linear simultaneous equations
.831	Determinants up to 3rd order
.861	Group
.8951	Addition of vectors
.8952	Multiplication by a scalar
.8953	Scalar product
.8954	Vector product
.8955	Triple products
.896	Matrices 2×2 ; 3×3
.8962	Inverse of a non-singular matrix 2×2
513.52	Similitude and homothety
.56	Homogeneous co-ordinate (*a*) Cartesian, (*b*) Triangular, (*c*) General
514.61	Right angles spherical triangle
.62	Spherical triangles in general
.65	Celestial sphere ; hour angle ; R.A. ; declination etc.
515	*Descriptive Geometry*
	Stereometry. Use of graphical and semi-graphical methods ; description geometry — solve problems of distance and angles
516.31	Distances and angles between planes ; 2 lines ; line and plane
.32	3 dimensional analytic geometry of plane and straight line
.501	Polar co-ordinates
.502	Graphs of well known curves, cycloids, epicycloids, hypocycloids, cardiods etc.
.503	Curve tracing ; asymptotes ; double point, curves inflexions etc., both for Cartesian equations, polar equations : co-ordinates in terms of a parameter
.507	General equation of 2nd degree in two variables
.552	Invariants associated with translations, rotations, reflexions, enlargements, sheers etc. — Products of transformation
517.11	$\lim F(n)\ n \rightarrow \infty$
.12	Series ; convergence
.13	Tests for convergence
.14	$\lim F(n)$; $n \rightarrow a$
.15	Continuity
.201	Partial differentiation
.210	Use of Taylor's and Maclaurin's theorems — assuming their existence
.211	Binomial series, logarithms, exponential series
.212	Existence of Taylor's series and the remainder after n terms
.213	Series for sin x, cos x
.215	Arithmetic geometric series

517.216 Summation of 'finite series'; induction
.217 Binomial theorems for positive integral exponent
.218 Relation between coefficients
.271 Rolle's theorem for polynomial functions, with proof
.272 Rolle's theorem in general (intuitive approach but precise statement)
.273 Maxima, minima, inflexions
.30 Definition of the definite integral and properties
.312 Techniques of integration
Substitution
Partial fractions
Parts
.313 Concept of integrating as the inverse of differentiating
.390 Areas assuming existence
.394 Length of arc
.395 Areas of surface revolution
.521 Logarithms and exponential functions
.522 Circular functions
.61 Numerical methods
.62 Solution of numerical equations, Newton. Morner etc.
519.1 Fundamental laws of probability

TOPICS SPECIFIC TO TURKEY

509 History of mathematics
511.10 Notational systems
512.05 Identities
.06 Partial fractions
.08 Inequalities (involving modification, squaring, square root : 2nd degree)
.241 Operations with complex numbers
.243 Solution of trig. equations involving all relevant angles
.810 Natural numbers
.8102 Extension to integers
.8103 Extension to rationals
.8106 Complex numbers
.812 Divisibility
.8121 Euclid's algorithm (not polynomials)
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.8951 Addition of vectors
.8952 Multiplication by a scalar
513.11 Triangle and associated circles ; Simson line ; Circle of Apollonius
.12 Ceva and Menelaus's theorems
.51 Coaxial circles and associated systems
.52 Similitude and homothety
.53 Inversion in a plane
.54 Reciprocation of w.r. to circle
.55 Duality : Pascal and Brianchon's theorems
.59 Involution systems on a straight line
516.22 Orthogonal projection — ellipse as projection of circle
.503 Curve tracing ; asymptotes ; double points, curves of inflexions etc. both from Cartesian equations, polar equations : co-ordinates in terms of a parameter
.506 Quadrilateral and quadrangular
.507 General equation of 2nd degree in two variables
.508 Family $S + kS' = 0$
.551 Pole and polar
.552 Invariants associated with translations, rotations, reflexions, enlargements, sheers etc. — Products of transformation
.56 Harmonic ranges and pencils treated matrically
.7 Differential geometry
.81 Position vector ; volocity vectors ; acceleration vector
.82 Uniform acceleration in a straight line
.83 Uniform acceleration in a plane ; parabolic motion
517.11 $\lim F(n)\ n \to \infty$
.14 $\lim F(n)\ n \to a$

517.15 Continuity
.215 Arithmetic geometric series
.216 Summation of 'finite series'; induction
.217 Binomial theorems for positive integral exponent
.273 Maxima, minima, inflexions
.293 Leibnitz's theorem on differentiation of product
.30 Definition and properties of the definite integral
.31 Methods of integration
.312 Techniques of integration
Substitution
Partial fractions
Parts
.313 Concept of integrating as the inverse of differentiating
.38 Differential equations

$$\frac{dy}{dx} = ky$$

$$P (y)\ dy = Q (x)\ dx$$

.390 Areas assuming existence
.391 Volumes of solids of revolution
.394 Length of arc
.395 Areas of surface revolution
.521 Logarithms and exponential functions
.522 Circular functions
531.1 *Kinematics*
Motion in a circle with uniform speed
Motion of a particle (heavy) constrained in a vertical circle
Tangential and normal components
Motion in a plane with acceleration towards a fixed point
.32 Simple harmonic motion
Simple sine wave motion
Combination of SHMs in a plane — equal frequencies

TOPICS SPECIFIC TO THE UNITED KINGDOM

512.05 Identities
.06 Partial fractions
.08 Inequalities (involving modification, squaring, square root : 2nd degree)
.241 Operations with complex numbers
.242 De Moivre's theorem
.243 Solution of trig. equations involving all relevant angles
.5 Combinatory analysis
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822 Conditions for consistency of up to 3 linear simultaneous equations
.831 Determinants up to 3rd order
.832 Higher order
.8951 Addition of vectors
8952 Multiplication by a scalar
.8953 Scalar product
.8954 Vector product
513.11 Triangle and associated circles ; Simson line ; Circle of Apollonius
.51 Coaxial circles and associated systems
.58 Properties of line pairs
516.22 Orthogonal projection — ellipse as projection of circle
.31 Distance and angles between planes ; 2 lines ; line and plane
.32 3 dimensional geometry of plane and straight line
.501 Polar co-ordinates
.503 Curve tracing ; asymptotes ; double points, curves inflexions etc., both from Cartesian equations, polar equations : co-ordinates in terms of a parameter
.505 Cross ratio
.507 General equation of 2nd degree in two variables
.508 Family $S + kS' = 0$
.551 Pole and polar
.7 Differential geometry

516.81 Position vector ; velocity vector; acceleration vector
.82 Uniform acceleration in a straight line
.83 Uniform acceleration in a plane ; parabolic motion
517.11 lim F (n) $n \to \infty$
.12 Series ; convergence
.14 lim F (n) $n \to a$
.17 Infinite integral. Convergence
.21 Series
.210 Use of Taylor's and Maclaurin's theorems — assuming their existence
.211 Binomial series, logarithms, exponential series
.213 Series for sin x, cos x
.214 Arc tan x
.215 Arithmetic geometric series
.216 Summation of 'finite series' ; induction
.217 Binomial theorems for positive integral exponent
.273 Maxima, minima, inflexions
.291 Curvature
.293 Leibniz's theorem on differentiation of product
.30 Definition and properties of the definite integral
.312 Techniques of integration

Substitution

Partial fraction

Parts

Reduction formulae

.313 Concept of integrating as the inverse of differentiating
.38 *Differential equations*

$$\frac{dy}{dx} = ky$$

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c = 0$$

$$\frac{d^2y}{dx^2} + n^2y = 0$$

$$\frac{dy}{dx} + Py = Q$$

$$P(y)\,dy = Q(x)\,dx$$

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

.390 Areas assuming existence
.391 Volumes of solids of revolution
.392 Centroids, moments of inertia
.394 Length of arc
.395 Areas of surface of revolution
.521 Logarithms and exponential functions
.522 Circular functions
.523 Hyperbolic functions
.524 Inverse circular functions
.525 Inverse hyperbolic functions
.61 Numerical methods
.62 Solution of numerical equations, Newton, Morner etc.
.63 Evaluation of definite integrals, Simpson, Gauss etc.
.64 Approx to functions by polynomials
519.1 Fundamental laws of probability
.8 Presentation ; pie charts, histograms, frequency, polygons
.900 Mean, mode, median, deciles, percentiles
.901 Measures of dispersion, variance
.902 Binomial distribution
.903 Poisson distribution
.904 Normal — Gaussian — distribution
905 Sampling : estimation of population parameters
.906 Tests for significance for small samples
.907 t test, F test, difference of means etc.

519.908 Bivariate normal distribution
.909 Regressive lines ; method of least squares
.910 Product moment correlation
.911 Ranking method
.913 Simple analysis of variance techniques
531.1 *Kinematics*
Motion in a circle with uniform speed
Motion of a particle (heavy) constrained in a vertical circle
Radial and transverse components of acceleration
Tangential and normal components
Motion in a plane with acceleration towards a fixed point
.32 Simple harmonic motion
Simple sine wave motion
Combinations of SHMs in a plane — equal frequencies

TOPICS SPECIFIC TO FINLAND

511.10 Notational systems
512.08 Inequalities (involving modification, squaring, square root : 2nd degree)
.241 Operations with complex numbers
.243 Solution of trig. equations involving all relevant angles
.810 Natural numbers
.8102 Extension to integers
.8103 Extension to rationals
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.8951 Addition of vectors
.8952 Multiplication by a scalar
513.1 Elementary plane geometry
.11 Triangle and associated circles
.52 Similitude and homothety
.56 Homogeneous co-ordinates (Cartesian)
516.22 Orthogonal projection — ellipse as projection of circle
.81 Position vector ; velocity vectors ; acceleration vector
517.12 Series ; convergence
.13 Tests for convergence
.14 lim F (n) $n \rightarrow a$
.215 Arithmetic geometric series
.216 Summation of 'finite series' ; induction
.217 Binomial theorems for positive integral exponent
.218 Relation between coefficients
.273 Maxima, minima, inflexions
.313 Concept of integrating as the inverse of differentiating
.391 Volumes of solids of revolution
.395 Areas of surface of revolution
.521 Logarithms and exponential functions
.522 Circular functions
.524 Inverse circular functions
531.1 *Kinematics*
Motion in a circle with uniform speed
Radial and transverse components of acceleration
Tangential and normal components
Motion in a plane with acceleration towards a fixed point

APPENDIX V

Time Allocation: Mathematics

(Replies to a questionnaire submitted to subject experts)

Can you state the number of lessons devoted to the syllabus in preparation for the terminal examination ? Please state :

(*a*) the number of years,
(*b*) the number of lessons per week.
(*c*) the number of teaching weeks per year.
(*d*) the length of lessons, on average, in minutes,
(*e*) the percentage of practical work in relation to the total length of the lessons.

AUSTRIA
(*a*) 8 years
(*b*) 4 lessons
(*c*) about 36 weeks per year
(*d*) 50 minutes

BELGIUM
(*a*) terminal class
(*b*) 7 lessons per week
(*c*) about 35 weeks per year
(*d*) 50 minutes

CYPRUS
(*a*) the examination syllabus is identical with the syllabus of the final year
(*b*) varies with subjects and sections

Section:	*Lessons per week:*
classical	4
agricultural	4
science	6
economic	3
technical	4

(*c*) 30 weeks per year
(*d*) 45 minutes

FRANCE
(*a*) 3 years
(*b*) 5+7+6 lessons per week
(*c*) 35 weeks per year
(*d*) 60 minutes
(*e*) 1/4 to 1/5 of time on practical work

FEDERAL REPUBLIC OF GERMANY
(*a*) final exam is not limited to subjects done in the last 2 years — so one cannot indicate the number of years devoted to the syllabus in preparation for the final exam
(*b*) 5 lessons per week (in last 2 classes)
(*c*) 40 weeks per year
(*d*) 45 minutes (sometimes 40)

GREECE
(*a*) 2 years
(*b*) 4 lessons per week
(*c*) 30 weeks per year
(*d*) 50 minutes

IRELAND
(*a*) 2 years
(*b*) 6 lessons per week
(*c*) 32 weeks per year
(*d*) 40 minutes

ITALY
(*a*) 5 years
(*b*) 5+4+3+3+3 lessons per week
(*c*) 34 weeks per year
(*d*) 60 minutes

LUXEMBOURG	(*a*) generally the last year — but it is understood that certain basic topics have been treated in the lower classes (*b*) 6 lessons per week in *section latine* 8 1/2 lessons per week in *section moderne* (*c*) 40 weeks per year (*d*) for 4 days a week the lessons last 55 minutes — the other two days the lessons last for 45 minutes
NETHERLANDS	(*a*) 5 to 6 years (*b*) in 5 years 5+5+5+6+5 (h. b. a) in 6 years 4+4+3+5+5 (gymnasium) (*c*) 38 to 39 weeks per year (*d*) 50 miuutes
NORWAY	(*a*) 3 years (*b*) 6+5+7 lessons per week (*c*) 38 weeks per year (*d*) 45 minutes
SWITZERLAND	(*a*) 4 years for final exam (1 to 2 for canton. exams) (*b*) 4 to 8 lessons per week (*c*) 40 weeks per year (*d*) 45 minutes
SWEDEN	New gymnasium, technical and scientific lines: (*a*) 3 years (*b*) 5+5+5 lessons per week (*c*) officially 39 weeks but about 6 may be discounted because of games, tests, Holy Days, etc. (*d*) 45 or 40 minutes
TURKEY	(*a*) the last year (*b*) year 1 / year 2 / year 3 — science 5 / 6 / 7; arts 5 / 4 / 3 (*c*) 32 weeks per year (*d*) 45 minutes
UNITED KINGDOM	(*a*) last 2 years (*b*) 7-8 (varies with school) (*c*) 39 (*d*) 40 minutes (*e*) varies with teachers
FINLAND	(*a*) 3 years (*b*) 5+5+5 lessons per week (*c*) 33 weeks per year (*d*) 45 minutes

APPENDIX VI

Specimen Timetables for Academic Secondary Schools showing time allocated to mathematics

A. **Belgium : Catholic**

General Timetable for Humanités anciennes *section*

Subject	Latin-Gk Class 3	Latin-Gk 2	Latin-Gk 1	Latin-Math. 3	Latin-Math. 2	Latin-Math. 1	Latin-Sc. 3	Latin-Sc. 2	Latin-Sc. 1
Religion	2*	2*	2*	2*	2*	2*	2*	2*	2*
French	3*	3*	3*	3*	3*	3*	3*	3*	3*
Latin	6	5*	5*	6	5*	5*	6	5*	5*
Greek	4	4	4	0	0	0	0	0	0
Gk. culture	0	0	0	0	0	0	0	1	1
2nd language	4	4	4	4	4	4	4	4	4
3rd language	1	1	1	1	1	1	1	1*	1*
4th language	0*	0*	0*	0*	0*	0*	0*	1*	1*
History	2	2	2	2	2	2	2	2	2
Maths.	3	3	3	7	8	9	4	4	4
Physics							2	2	3
Chemistry	1*	2	2	1*	2	2	1	1	1
Biology							1	1	1
Geography	1*	1	1	1*	1	1	1*	1	1
Phys. Ed.	1*	1*	1*	1*	1*	1*	1*	1*	1*
Aesthetics	0	0*	0*	0	0*	0*	0	0*	0*
Drawing	0*	0	0	1*	1*	1*	0*	0*	0*
Music	0*	0	0	0*	0	0	0*	0	0
Total	28	28	28	29	30	31	28	29	30

Belgium : Catholic

General Timetable for Humanités modernes *section*

Subject	Scient. A Class 3	Scient. A 2	Scient. A 1	Scient. B 3	Scient. B 2	Scient. B 1	Econom. 3	Econom. 2	Econom. 1
Religion	2*	2*	2*	2*	2*	2*	2*	2*	2*
French	5*	5*	4*	5*	5*	4*	5*	5*	4*
Ancient culture	0	0	0	0	1	1	0	0	0
2nd language	4*	4*	4*	4*	4*	4*	4*	4*	4*
3rd language	3	2	2	3	3	2	3	2	2
4th language	0*	0*	0*	0*	0*	0*	3	3	2
History	2	2	2	2	2	2	2	2	2
Maths.	7	8	9	5	5	5	3	3*	4
Ec. science	0	0	0	0	0	0	4	5	6
Physics				2	2	3			
Chemistry	2	2	2				2	2	2
Biology				3	3	3			
Science Lit.	0	0	0	0	0	1	0	0	0
Geography	1*	1	1	1*	1	1	1*	1	1
Phys. Ed.	1*	1*	1*	1*	1*	3*	1*	1*	1*
Aesthetics	0	0*	0*	0	0*	0*	0	0*	0*
Drawing	1*	1*	1*	1*	1*	1*	0	0	0
Music	0*	0	0	0*	0	0	0*	0	0
Total	28	28	28	29	30	30	30	30	30

* The asterisk means that a lesson of 50 mins. can be added to the subject timetable provided that the lesson time does not exceed 32 hours per week.

B. SWEDEN: *The Gymnasiet*

1. The duration of Gymnasium studies is normally three years. except for engineering when it is four years.

2. Specialisation is introduced late and by degrees; definite lines of study do not occur until the second year.

3. There are five lines of study: (*a*) liberal arts (humanities) [Hum]
(*b*) social sciences [Sh]
(*c*) economics [Ek]
(*d*) natural sciences [Na]
(*e*) technology [Te]

4. Further specialisation may be made within the lines of study, e. g. there are four branches in the economics line and six in the technology line.

General construction of the curriculum and timetable

There are 34, 32, and 30 lessons a week in forms 1 to 3 respectively. These include 'lessons available' for various special tasks such as lectures, study visits and individual guidance. The first form is, generally speaking, the same for all students. New or radically revised subjects are:

1. General linguistics
2. History of art and music
3. Psychology
4. Philosophy
5. Nature studies
6. Economics of business
7. Technology and Ergonomics

Total number of lessons a week in grades 10-12 (13) in different subjects included in the five lines of the *gymnasiet*.

Subject			Ek			Te	
	Hum	Sh	Esp	Oth	Na	1-3	4
Swedish	10	10	9	9	9	8	
Modern languages	30	25	30	21	18	12	
General linguistics	3						
History	8	8	4	4	6	4	
History of music and art	2	2			2		
Religion	3	3	2	2	2	2	
Philosophy	2	2			2		
Psychology	2	2	2	2	1		2c
Civics	$10^1/_2$	$10^1/_2$	$8^1/_2$	$8^1/_2$	5	5	
Law			2b	2			
Economics of business			10	13			3
Other econ. subjects				7			
Mathematics	5a	11	5	11	15	15	
Physics					$10^1/_2$	$10^1/_2$	

			Ek			Te	
	Hum	Sh	Esp	Oth	Na	1-3	4
Chemistry							7
Biology							5
Nature studies		7		9	3a	3a	
Technology							
Other tech. subjects							
Typewriting					4	3	
Shorthand					6		
Music or art	2	2			2		
Gymnastics	8	8	7	7	8	7	
Available	3½	3½	3½	3½	3½	3½	
Total	96	96	96	96	96	96	35

a = free choice between two courses
b = May be exchanged for practical or secretarial work
c = Ergonomics

C. LUXEMBOURG

Secondary Education (for boys)

Higher Division

Subjects	III			II				I			
	GL	A	B	GL	A	B	C	GL	A	B	C
R. I.	2	2	2	2	2	2	2	2	2	2	2
French	5	5	5	4	4	4	4	4	4	4	4
English	2	4	4	2	4	4	4	2	4	4	4
German	3	3	3	3	3	3	3	3	3	3	3
Latin	6	6	5	6	6	4	4	6	6	3	4
Greek	4			4				4			
Greek authors		1							1		
Mathematics	3	3	5	3	3	5	3	3	3	6	3
Special maths.							1				1
Geom. drawiug			2			1					
Gen. and nat. History	2	2	2	2	2	2	2	2½	2½	2½	2½
Geography	1	1	1								
Natural science	2	2	2	2	2	2	2				1
Practical					1		1				
Physics				2½	2½	2½	2½	2½	2½	2½	2½
Practical						½	½			½	½
Chemistry				1½	1½	1½	1½	1½	1½	1½	1½
Practical						½	½			½	½
Public law and admin.								½	½	½	½
Art	1	1		1	1		1	1	1		1
P. E.	1	1		1	1	1	1	1	1	1	1
Music											
	30	31	32	30	31	32	32	30	31	31	31
	33	32	33	33	32			33	32		32

Abbreviations :
GL = Latin Greek section
A = Latin A section (language bias)
*B = Latin B section (science bias)
C = Latin C section (natural science, medicine bias)

D. DENMARK

Mathematics Line

Special subjects

	Common subjects Years I	II	III	*Mathematics Physics Years II	III	Civics branch Years II	III	Natural science Years II	III
Religion	1	1	1						
Danish	4	4	4						
English / German	4	3	0						
French (Russ.)	5	3	4						
Latin									
Greek with Cl. Antiquities									
Cl. Antiquities	1	1	1						
History	4	3	3						
Civics				0	1	5	5	0	1
Geography	2	2							2
Biology				0	4	0	4	4	7
Biochemistry									
Chemistry	2			2	2	1	1	2	2
Physics	3			5	4	3	2	3	2
Mathematics	5			6	6	4	3	4	3
	31	17	13	13	17	13	17	13	17
Physical Ed.	3	3	3						
Common Periods	0	1	1						
Music max.	2	2	2						
min.	(2)	(0)	(0)						
Art and Art Appreciation	(0)	(2)	(2)						
	36	23	19	13	17	13	17	13	17

In all, 36 periods a week.

E. NETHERLANDS

Time-table of the *hogereburgerschool* (h.b.s) division A and B, indicating the number of lessons (of 50 minutes) per week in each school year

Secondary-school years:	I	II	III	IVA	IVB	VA	VB	Total A	Total B
Dutch	4	4	4	4	3	4	3	20	18
French	5	3	3	4	3	4	2	19	16
English	3	3	3	4	2	4	2	17	13
German	—	3	3	4	2	4	2	14	10
Commercial science	—	—	1	1	—	6	(2)	14	3 (1)
Political economy	—	—	—	2	—	2	1	4	1
Geography	3	3	2	2	2	2	1	12	11
History	4	2	2	2	2	3	2	13	12
Civics	—	—	1	1	—	1	—	3	1
Mathematics	5	5	5	—	5	—	5	15	25
Mechanics	—	—	—	—	2	—	2	—	4
Physics	—	2	3	—	3	—	3	5	11
Chemistry	—	—	2	2	4	2	4	6	10
Biology	2	2	—	—	2	—	2	4	8
Cosmography	—	—	—	—	—	—	1	—	1
Hand drawing	3	2	1	—	1	—	(1)	6	8 (7)
Linear drawing	—	—	—	—	1	—	(1)	—	2 (1)
Physical education	3	3	3	2	2	2	2	13	13
Total	32	32	33	34	34	34	34	165	167 (165)

pupils of VA can choose between commercial science and drawing

Time-table of the gymnasium (division A and B)

Secondary-school years	I	II	III	IV	VA	VB	VIA	VIB	Total A	Total B
Greek	—	5	5	5	6	3	8	3	29	21
Latin	7	5	5	5	7	3	8	3	37	28
Dutch	4	3	3	3	2	2	3	3	18	18
French	4	3	3	2	2	2	2	2	16	16
German	—	—	3	2	2	2	2	2	9	9
English	—	3	2	2	2	2	2	2	11	11
History	4	3	2	2	4	3	4	2	19	16
Geography	3	2	1	2	1	1	—	—	9	9
Mathematics	4	3	3	3	2	5	2	5	17	23
Physics	—	—	2	2	—	3	—	3	5	10
Chemistry	—	—	—	2	3	3	—	4	3	9
Biology	2	2	—	—	—	2	—	2	5	8
Physical education	3	3	3	2	2	2	2	2	15	15
Drawing	2	1	1	1	—	—	—	—	5	5
Total	33	33	33	33	33	33	33	33	198	198
Hebrew					2	2	2	2		

APPENDIX VII

Projected general changes in the mathematics syllabus

INTRODUCTION OF MODERN MATHEMATICS TOPICS

The figures in the tables indicate the number of years it will take for the topic to be introduced into the syllabus of that country. The ticks indicate that the topic is already part of the syllabus.

		Austria	Cyprus	Greece	Ireland	Luxembourg	Netherlands	Norway	United Kingdom
509	History of mathematics	x	x	x	+5				
511.11	Binary arithmetic (application to computers)	5	4	4	X		8		2
512.06	Partial fractions		x	x	+5	x		x	x
.08	Inequalities (involving modification, squaring. square roots, 2nd degree)	5	x	x	x	x	x	x	x
.241	Operations with complex numbers	x	x	x	x	x	8	x	x
.242	De Moivre's theorem	x	x	x	x	x	8	x	x
.243	Solution of trig. equations involving all relevant angles	x	x	x	x	x	8	x	x
.5	Combinatory analysis	x	3	3	x	x		x	x
.800	Set; subset	5	2	2	x	x	8		
.801	Inclusion, intersection, union	5	2	2	x	x	8		
.802	Empty set	5	2	2	x	x	8		
.803	Complementation	5	2	2	x	x	8	7-8	
.804	Equivalence relations; order relations		2	2	x	x	8		
.805	Boolean algebra		4	4	+5				
.806	Mapping, function	5	4	4	x	x	8		
.807	Many - one		4	4	x	x	8		
.808	One - one		4	4	x	x	8		
.809	Vector space	5		4	+5	1	8		2
	Neighbourhood				+5	2			2
	Topology and topological spaces	5			+5	2			
.81	Theory of numbers		4	4					
.8101	Peano's Axioms	5	x	x				x	
.8104	Properties of real numbers		x	x	x	2	x	x	

		Austria	Cyprus	Greece	Ireland	Luxembourg	Netherlands	Norway	United Kingdom
.8105	Dedekind section (or any alternative construction of real numbers)	5	x	x				x	
.8106	Complex numbers	x	x	x	x	x	8		
.812	Divisibility							x	
.8121	Euclid's algorithm (not polynomials)	x	x	x		x	8	x	
.8122	Congruences	5					8		
.815	Algebraic fields	5	4	4	+5				2
.8151	Properties of a field, ring	5	4	4	+5	x			2
.822	Conditions for consistency of up to 3 linear simultaneous equations	x	x	x	+5	x	x	x	x
.83	Determinants		2		+5		7-8	7-8	
.831	Determinants up to 3rd order	5	2	x		x		x	
.832	Higher order		x	x					x
.86	Theory of groups	5			+5		8		
.862	Euclidean geometry as study of invariants under group transformations					x	8		
.895	Vector algebra	5	2	2				7-8	
.8951	Addition of vectors	5	2	2	x	x	x	x	x
.8952	Multiplication by a scalar	5	2	2	x	x	x		x
.8953	Scalar product	5	2	2	x	x	8		x
.8954	Vector product		2	2	x	x			x
.8956	Differentiation		x	x					2
.8958	Operator		x	x					
513.3	Mensuration of pyramid, cone, sphere (volume and surface)	2							

		Austria	Cyprus	Greece	Ireland	Luxembourg	Netherlands	Norway	United Kingdom
.5	Modern geometry (post-Euclidean)							7-8	
.52	Similitude and homothety	x			+5	x	x		
.53	Inversion in plane	5			x				
.56	Homogenous co-ordinates (*a*) Cartesian	5							
.57	Line co-ordinates	5							
516.31	Distances and angles between planes: 2 lines, line and plane	x			+5		8		x
.32	3 dimensional and analytical geometry of plane and straight line	5			+5		8	7-8	x
.503	Curve tracing; asymptotes, double points, curves inflections etc. both from Cartesian equations; polar equations; co-ordinates in terms of a parameter	5			x				x
.505	Cross ratio	5			x				x
.552	Invariants associated with translations, notations, reflexions, enlargements, sheers etc. Products of transformation	5				x	8		2
.56	Harmonic ranges and pencils (faisceaux) deferred matrically	5			x				
.81	Position vector; velocity vectors; acceleration vector				x		7-8	7-8	
517.13	Tests for convergence	5	x	x	x	x			
.201	Partial differentiation	5			+5				
.210	Use of Taylor's and Maclaurin's theorems assuming their existence	5				x			x
.211	Binomial series, logarithms, exponential series	5			x	x			x
.212	Existence of Taylor series and remainder after n terms	5			+5				

		Austria	Cyprus	Greece	Ireland	Luxembourg	Netherlands	Norway	United Kingdom
.213	Series for sin x, cosine x	5			+5	x			x
.27	Variation of function values				+5				
.312	Techniques of integration. Substitution	x			+5	x		x	x
	Partial fractions					x			x
	Parts	5				x			x
	Reduction formulae				x				x
.38	Differential equations	5							
	$\frac{dy}{dx} = ky$				x				x
	$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c = 0$					1	8		x
	$\frac{d^2y}{dx^2} + n^2y = 0$				x				x
	$\frac{dy}{dx} + Py = 0$								x
	$P(y)\,dy = Q(x)\,dx$								x
	$\frac{d^2v}{dx^2} + a\frac{dy}{dx} + by = f(x)$								x
.524	Inverse circular functions	5	x	x	x	x			x
.61	Numerical methods	5			x				x
.62	Solution of numerical equations, Newton, Morner etc.	5			x				x
.63	Evaluation of definite integrals. Simpson, Gauss etc.	5							x
.64	Approx to functions by polynomials	5							
519.1	Probalities	x							
.	Fundamental laws of probability	5	x	4	x	x	8		x

		Austria	Cyprus	Greece	Ireland	Luxembourg	Netherlands	Norway	United Kingdom
.8	Treatment of data								
	Presentation, pie charts, histograms ; frequency, polygons		x	4	x		8		x
.900	Mean, mode, median, deciles, percentiles	5	4		x		8		x
.901	Measures of dispersion, variance	5			x		8		x
.902	Binomial distribution	5			x		8		x
.903	Poisson distribution	5							x
.904	Normal (Gaussian) distribution				x		8		x
.905	Sampling : estimation of population	5							x
.911	Ranking methods								x
.92	Simple linear programmes	5			x				2

APPENDIX VIII

List of the official, semi-official and recommended books on teaching methods

(Replies to a questionnaire)

AUSTRIA : There are no official books, but there is a periodical which deals with pedagogical problems : *Erziehung und Unterricht*.

BELGIUM : Ministère de l'Education Nationale. Pamphlets and papers are published by the 'Conseil de Journées d'Etudes'.

CYPRUS : There are no official books — teachers are free to use whichever methods they deem appropriate. Guidance is given by the Inspectorate.

FRANCE : There are no official documents on teaching methods as such.

FEDERAL REPUBLIC OF GERMANY : *Richtlinien für den Unterricht in der Höheren Schule : Mathematik* also gives recommendations on teaching methods.

GREECE : There are none. Every teacher can use the appropriate method under the guidance and supervision of the Ministry of Education.

IRELAND : There are none.

ITALY : There are no official books ; the teacher is free to use appropriate methods.

LUXEMBOURG : There is no official book. German methodological works are used in teacher training colleges. Didactic questions are discussed in the *Journal des Professeurs*.

NETHERLANDS : There are none.

NORWAY : Curriculum and syllabuses are issued by the Ministry of Education.

SWITZERLAND : There are none.

SWEDEN : Recommendations for teaching method are contained in *Lüroplan för gymnasiet*.

TURKEY : There is no official book on the methods of instruction. A semi-official organisation supported by the Ministry has arranged for the translation of the following books :

(1) *Method and Models in Modern Mathematics*, Cogan, Kemeny, *et all.*
(2) *L'enseignement des mathématiques dans les pays de l'OCDE.*
(3) *New Thinking in School Mathematics (OECD).*
(4) *Mathématiques dans les pays de l'OCDE.*

UNITED KINGDOM : Recommended books are :

(1) *Teaching of Mathematics in Secondary Schools*, HMSO.
(2) *Curriculum Bulletin No. 1, The Schools Council*, HMSO.
The Mathematical Association
The Teaching of Sets in Schools (K.R McLean).
The Teaching of Algebra in Sixth Forms.
The Teaching of Higher Geometry in Schools.
The Teaching of Calculus in Schools.
The Teaching of Trigonometry in Schools.
Analysis Course I.
The Use of Visual Methods in Teaching Mathematics.
A Second Report on the Teaching of Mechanics in Schools.

FINLAND : Teachers use corresponding publications in German and in English.

APPENDIX IX

The order of the syllabus

(Replies to a questionnaire)

Is an order for the teaching of the syllabus prescribed ?

AUSTRIA :	No.
BELGIUM :	No.
CYPRUS :	No.
FRANCE :	No.
FEDERAL REPUBLIC OF GERMANY :	There are compulsory topics for each class but the teacher is free to choose the order in which they are treated.
GREECE :	The order is prescribed in the curriculum guide of the Ministry of Education.
IRELAND :	Not formally.
ITALY :	It is necessary to complete the prescribed syllabus every year.
LUXEMBOURG :	Syllabuses prescribe the order in which the topics are to be treated. If the course has distinct parts (algebra, geometry, trigonometry) the teacher is free to treat them simultaneously or successively.
NETHERLANDS :	No.
NORWAY :	Curriculum and Syllabus are issued by the Ministry of Education.
SWITZERLAND :	No.
SWEDEN :	Yes : the syllabus has to be taught in the order given in *Läroplan för gymnasiet*. This is necessary as the students must take part in the tests given by the Board of Education in both the 2nd and 3rd year of studies (in the upper secondary school). It is stated what stage studies must have reached.
TURKEY :	Certain topics have to be taught in certain years. (No indication of order with the syllabus for the year.)
UNITED KINGDOM :	No.
FINLAND :	The National Board of Schools *(Kouluhallitus)* has confirmed the methodological directives for the teaching of all subjects ; the directives are sent to all schools in connection with the syllabuses.

APPENDIX X

Changes in teaching method

(Replics to a questionnaire)

Describe briefly any changes in teaching methods contemplated (e.g. use of teaching machines, language laboratories, re-appraisal of the role of practical work).

AUSTRIA :	None.
BELGIUM :	None.
CYPRUS :	Teaching methods have been moving in the last few years to the more active participation of the pupil.
FRANCE :	No reply can be given at the moment.
FEDERAL REPUBLIC OF GERMANY :	Experiments in teaching method have started.
GREECE :	None.
IRELAND :	None.
ITALY :	No changes in teaching method.
LUXEMBOURG :	No change.
NETHERLANDS :	None.
NORWAY :	None.
SWITZERLAND :	None.
SWEDEN :	No comment.
TURKEY :	In the science *lycée* the 'discovery method' is frequently used in teaching mathematics.
UNITED KINGDOM :	The Nuffied Project for Primary School Mathematics with an emphasis on 'Discovery Learning' is being extended to 11-13 age group and may be extended still further. The British Broadcasting Corporation is conducting a series on 'Mathematics Teaching' television programmes which are being used gradually by more and more schools. The Mathematics Association has a committee which is gathering material for a report on the function of the mathematical laboratory in schools — more new schools are being built with a room designed especially for this purpose, and many existing schools are converting a room for the same purpose. The use of calculating machines in mathematics teaching is growing slowly.
FINLAND :	Interest has been shown in programmed learning and minor experiments are going on in the field of mathematics.

APPENDIX XI

Nature of the academic secondary leaving examination in relation to mathematics

AUSTRIA

1. Can you list the subjects which are studied in the last two years of the secondary school ?	Religion, German (mother tongue). 2 modern languages (usually English and French), history, geography, biology, physics, chemistry mathematics, descriptive geometry, introduction into philosophy, music or art (drawing), physical training.
2. Can you list the subjects which are examined by a written paper and form part of the terminal examination ?	German, a modern language, mathematics, descriptive geometry.
3. How many written papers are there in mathematics ? — in the other examinable subjects ?	One examination paper per subject.
4. How much time is allowed for the mathematics examination ?	4 hours.
5. How many questions are there in the mathematics examination paper or papers ?	4 questions.
6. Is there any choice ? If so, how many questions must the pupil do ?	There is no choice.
7. What sort of questions are set ? e.g. " Questions de cours ", " Problems " ?	Problems. (Essay questions similar to the questions listed in Document CCC/EGT (66) 22, page 34-37.)
8. Is there an oral examination in mathematics ? Is it compulsory or does it depend on the pupil's performance in the written examination ?	Every student has to select 3 subjects for his oral examination. One of these subjects can be mathematics. But if a student has failed in the written examination in mathematics, he has to take an additional oral examination. There is no compulsory examination in mathematics.
9. How long is the oral examination ? Is any time allowed for preparation ?	The oral examination takes about 15 to 30 minutes per subject. The time for preparation is about 30 minutes.
10. Who are the examiners ? (*a*) for the written examination (*b*) for the oral examination	The examiner is in both cases the teacher who taught the student in the subject in the last year of the academic secondary school. He proposes the marks which are fixed by an examining board consisting of a supervisor, the headmaster and the teachers of the student.

11. Who prepares the examination ?	The teacher who taught the student in the last year of the secondary school. He works out two tests for the written examination. each consisting of four questions. One of these tests is selected by a supervisor for the examination. This teacher also prepares three questions for the oral examination. Two of these questions are selected by the chairman of the examining board.
12. Is the pupil's school record taken into account at all ?	Yes. it is. but not in all cases.

FRANCE

1. The subjects studied in the last 2 years	Philosophy, Physics, Mathematics, Chemistry, French, Mod. Languages, Latin, Greek, Music, History, Geography, Biology, Drawing, P.E.
2. The subjects which are examined by a written paper and which form part of the terminal examination	Philosophy, Mathematics, Physics, Biology, Modern Languages, History-Geography
3. How many written papers are there in mathematics ? — in other subjects ?	1 paper in each.
4. What is the time allocation for the mathematics paper ?	3 hours.
5. How many questions are on the paper ?	1
6. Is there any choice ? How many must the pupil answer from the total number ?	No.
7. What sort of questions are set ? e.g., " Questions de cours ", " Problems " ?	Problems.
8. Is there an oral ? Is it compulsory or does it depend on performance in the written examination ?	No *(oral de contrôle* for border-lines) (till 1968). Yes : from 1968.
9. (*a*) time allocation for oral (*b*) time allocation for preparation	N.A.
10. Who are the examiners ? (*a*) for the written examination (*b*) for the oral examination	Teachers for both.

11. Who prepares the examination?	The teachers submit questions to the Ministry: one is chosen from those submitted.
12. Is the pupil's school record taken into account at all?	Yes, if he gets a low mark in the examination.

FEDERAL REPUBLIC OF GERMANY

1. The subjects which are studied in the last 2 years	Religion Philosophy German History Social Studies Geography Music Modern languages (2) Mathematics Physics Chemistry Biology Handwork P.E.
2. The subjects which are examined by a written paper and which form part of the terminal examination	*All* German Social Studies R.I., P.E. *Science* Mathematics Physics Modern lang. + 1 subject (at choice)
3. How many written papers are there in mathematics? — in other subjects?	1 paper in each.
4. What is the time allocation for the mathematics paper?	5 hours.
5. How many questions are on the paper?	1 which may be subdivided into 3 or 4 parts.
6. Is there any choice? How many must the pupil answer from the total number?	Not usually.
7. What sort of questions are set? e.g., "Questions de cours", "Problems"?	
8. Is there an oral? Is it compulsory or does it depend on performance in written paper?	Number of orals that each pupil has depends on his performance.
9. (*a*) time allocation for the oral (*b*) time allocation for preparation	(*a*) 15-20 minutes. (*b*) 30 minutes.

10. Who are the examiners ? (*a*) for the written examination (*b*) for the oral examination	(*a*) Teachers — paper is circulated to the Board. Inspector reviews paper of prospective oral candidates. (*b*) Teachers, head, inspector.
11. Who prepares the examination ?	The teachers submit questions to the Ministry : one is chosen from those submitted.
12. Is the pupil's school record taken into account at all ?	Yes. Grade for each subject is based on written paper, oral examination, reports.

GREECE

1. The subjects which are studied in the last 2 years	Religion Ancient Greek Modern Greek Mathematics History Geography Astronomy Natural Sciences (physics, chemistry, biology) Psychology and Logic Drawing English or French P.E.
2. The subjects which are examined by a written paper and which form part of the terminal examination	All subjects are examined by a written paper except P.E.
3. How many written papers are there in mathematics ? — in other subjects ?	In mathematics there are 4 papers : (1) algebra. (2) geometry, (3) trigonometry, (4) descriptive geometry. All other subjects have one paper each except natural sciences which has three : (1) physics, (2) chemistry, (3) biology.
4. What is the time allocation for the mathematics paper ?	2 hours for each of the 4 papers.
5. How many questions are on the paper ?	3 questions on each paper.
6. Is there any choice ? How many must the pupil answer from the total number ?	Yes. 2 questions out of 3.
7. What sort of questions are set ? e.g., " Questions de cours ", " Problems " ?	Usually problems.
8. Is there an oral in mathematics ? Is it compulsory or does it depend on performance in written paper ?	No. Oral recitation is held in class during the year.
9. (*a*) time allocation for the oral (*b*) time allocation for preparation	None.
10. Who are the examiners ? (*a*) for the written examination (*b*) for the oral examination	The student's regular teachers.

11. Who prepares the examination ?	Each teacher prepares his own examination.
12. Is the pupils school record taken into account at all ?	Yes, the final grade is an average of the examination score and the daily performance score.

This final secondary examination does not entitle the student to enter the university. The candidate for the university in mathematics and science must take special written examinations in 5 subjects : composition, Ancient Greek, history, mathematics, physics-chemistry-biology (mathematics and physics are examined in 3 papers each). Successful examinees get the 'academic certificate' giving them the right to enrol at the corresponding Higher Institution.

IRELAND

1. The subjects which are studied in the last 2 years	Student must study 5 subjects in their last 2 years. Irish is compulsory — they choose the other 4 from some 25 subjects (among these are mathematics and applied mathematics).
2. The subjects which are examined by a written paper and which form part of the terminal examination	All 5 subjects (separate Pass and Honours syllabuses and examinations).
3. How many written papers are there in mathematics ? — in other subjects ?	2 in mathematics 1 in applied mathematics 2 in Irish, English, commerce and art 1 paper in all other subjects (Pass and Honours).
4. What is the time allocation for the mathematics paper ?	2 1/2 hours for each paper.
5. How many questions are on the paper ?	10 on each paper. 9 in applied mathematics.
6. Is there any choice ? How many must the pupil answer from the total number ?	Yes, 6 out of 10. Up to 1965 there was little or no choice e.g. 6 out of 7, but with the introduction of revised syllabuses a wider choice is being given for a transitional period.
7. What sort of questions are set ? e.g.. "Questions de cours", "Problems" ?	A mixture ; but few of the questions would be mere reproduction.
8. Is there an oral in mathematics ? Is it compulsory or does it depend on performance in written examination ?	No.
9. (*a*) time allocation for the oral (*b*) time allocation for preparation	
10. Who are the examiners ? (*a*) for the written examination (*b*) for the oral examination	Department Inspectors are the examiners.

11. Who prepares the examination ?	Papers are set by Department of Education Inspectors who also supervise the marking. The actual work of marking is carried out by a team of teachers. The papers are submitted in advance to the University for acceptance of standard for exemption from university matriculation examination.
12. Is the pupil's school record taken into account at all ?	No.

There are two courses for the certificate ; one is a Pass course, the other an Honours course. In the Pass course a pupil must gain 40 per cent to pass on a Pass paper and 30 per cent for a pass on an Honours paper ; in the Honours paper a pupil must gain 60 per cent for Honours ; he must have Honours in three of five subjects.

Pass Certificate is awarded to pupils :

(i) who pass in Irish and 4 subjects

(ii) who obtain honours in 3 subjects and a pass in a 4th (Irish being one of 4).

Honours Certificate is awarded to pupils :

(i) who pass in at least 5 subjects (Irish) and have honours in 3

(ii) who obtain honours in 4 subjects (Irish).

NETHERLANDS HBS

1. The subjects which are studied in the last 2 years	Dutch French English German Political Econ. Geography History P.E.	Mathematics Mechanics Physics Chemistry Biology Cosmography Drawing
2. The subjects which are examined by a written paper and which form part of the terminal examination	Mathematics Physics Chemistry Dutch	French English German
3. How many written papers are there in mathematics ? — in other subjects ?	4 : algebra, geometry, trigonometry, descriptive geometry.	
4. What is the time allocation for the mathematics paper ?	2 1/2 hours for each paper.	
5. How many questions are on the paper ?	3 or 4.	
6. Is there any choice ? How many must the pupil answer from the total number ?	No.	
7. What sort of questions are set ? e.g., "Questions de cours", "Problems" ?	Problems.	

8. Is there an oral ? Is it compulsory or does it depend on performance in the written paper ?	Yes in Dutch, French, English, German, Mathematics, Physics, Chemistry, Biology, Geography, History, Political Econom. In Mathematics, Physics and Chemistry, if the mark obtained in the written test is 7 or above, there is no oral.
9. (*a*) time allocation for the oral (*b*) time allocation for preparation	(*a*) 40 minutes for Mathematics, for all other subjects, 20 minutes (*b*) For language orals, 15 minutes' preparation is allowed.
10. Who are the examiners ? (*a*) for the written examination (*b*) for the oral examination	(*a*) The written examination takes place under the supervision of the teaching staff (*b*) The oral examination is conducted by the senior teacher of the subject in question.
11. Who prepares the examination ?	The State School-Inspectors.
12. Is the pupil's school record taken into account at all ?	The school record is taken into account in two cases : (*a*) if the average mark for the school year is 6 or above, no oral examination is needed in Political Economy, Geography or History (*b*) The school record is available for consultation by the *deskundigen* (official observers of the examination appointed by the Minister of Education and Sciences).

APPENDIX XII

List of topics and sub-topics prescribed for study in constituent states of the Council of Europe

(A modification of the Universal Decimal Classification System is used)

509 History and local treatment of mathematics
History of mathematics
511 Arithmetic
.1 Numeration systems
.10 Notational systems
.11 Binary arithmetic (application to computers)
512 Algebra
.01 Analysis - algebra, calculus
.02 Polynomials
.03 Rational functions
.04 Graphs of such functions
.05 Identities
.06 Partial fractions
.07 Relation between roots of $P(z) = 0$ (2nd degree)
.08 Inequalities (involving modification, squaring, square root: 2nd degree)
.2 Algebraic equations and imaginary quantities
.21 Algebraic equations
.24 Imaginary quantities
.241 Operations with complex numbers
.242 De Moivre's theorem
.243 Solution of trig. equations involving all relevant angles
.5 Combinatory analysis
.8 Abstract algebra
.800 Set; subset
.801 Inclusion, intersection, union
.802 Empty set
.803 Complementation
.804 Equivalence relations; order relations
.805 Boolean algebra
.806 Mapping, function
.807 Many - one
.808 One - one
.809 Vector space
Neighbourhood
Topology and topological spaces
.81 Theory of numbers
.810 Natural numbers
.8101 Peano's Axioms
.8102 Extension to integers
.8103 Extension to rationals
.8104 Properties of real numbers
.8105 Dedekind Section (or any alternative construction of real numbers)
.8106 Complex numbers
.812 Divisibility
.8121 Euclid's algorithm (not polynomials)
.8122 Congruences
.815 Algebraic fields
.8151 Properties of a field, ring
.82 Theory of equations
.821 Simultaneous equations (3 linears, 1 linear, 1 2nd degree)
.822 Conditions for consistency of up to 3 linear simultaneous equations
.83 Determinants
.831 Determinants up to 3rd order
.832 Higher order
.833 Product of determinants

512.86 Theory of groups
.861 Group
.862 Euclidean geometry as study of invariants under group transformations
.895 Vector algebra
.8951 Addition of vectors
.8952 Multiplication by a scalar
.8953 Scalar product
.8954 Vector product
.8955 Triple products
.8956 Differentiation
.8957 Vector from a scalar field
.8958 Operator Δ
.896 Matrices 2×2; 3×3
.8961 Higher order
.8962 Inverse of a non singular matrix 2×2

513 Elementary Euclidean Geometry
.1 Elementary plane geometry
.11 Triangle and associated circles; Simson line; Circle of Apollonius
.12 Ceva and Menelaus's theorems
.3 Elementary solid geometry
.31 Mensuration of pyramid, cone, sphere (volume and surface)
.5 Modern geometry (post-Euclidean)
.51 Coaxial circles and associated systems
.52 Similitude and homothety
.53 Inversion in plane
.54 Reciprocation of w. r. to circle
.55 Duality: Pascal and Brianchon's theorems
.56 Homogenous co-ordinates (*a*) Cartesian
(*b*) triangular
(*c*) general
.57 Line co-ordinates
.58 Properties of line pairs
.59 Involutions systems on a straight line
.591 General homographies — line at infinity; circular points

514 Trigonometry
.5 Plane trigonometry
.51 Solution of triangles, sine rule, cosine rule etc.
.6 Spherical trigonometry
.61 Right angles spherical triangle
.62 Spherical triangles in general
.63 Geo-sphere: great circle navigation
.64 Map projection
.65 Celestial sphere; hour angle; R. A.; declination etc.
.66 Equation of time
.67 Solid angle

515 Descriptive geometry
Stereometry; Use of graphical and semi-graphical methods; description geometry — solve problems of distance and angles
.4 Conical projections
Real conical projections

516 Analytic Geometry
.1 Straight lines and co-ordinate axes
.11 Analytical geometry of straight line and circle
. Curves in Cartesian form
.21 Focus and directrix property of parabola, ellipse, hyperbola
.22 Orthogonal projection — ellipse as projection of circle
.23 Analytical geometry of parabola
.24 Analytical geometry of ellipse hyperbola referred to principal axes and rectangular hyperbola to asymptotes
.3 Plane and lines in space with Cartesian form
.31 Distance and angles between planes; 2 lines, line and plane
.32 3 dimensional analytic geometry of plane and straight line
.5 Modern algebraic geometry

516.501 Polar co-ordinates
.502 Graphs of well known curves, cycloids, epicycloids, hypocycloids, cardiods etc.
.503 Curve tracing; asymptotes; double points, curves inflexions etc. both from Cartesian equations, polar equations: co-ordinates in terms of a parameter
.504 Pedal (p,r) equations ; equations ; equations of the pedal etc.
.505 Cross ratio
.506 Quadrilateral and quadrangular
.507 General equation of 2nd degree in two variables
.508 Family $S + kS' = 0$
[illegible] Geometric transformat[illegible]
.551 Pole and polar
.552 Invariants associated with translations, rotations, reflexions, enlargements, sheers etc. Products of transformation
.56 Harmonic ranges and pencils (faisceaux) deferred matrically
.7 Differential geometry
8 Kinematic geometry
.81 Position vector; velocity vectors; acceleration vector
.82 Uniform acceleration in a straight line
.83 Uniform acceleration in a plane: parabolic motion
.84 Hodograph

517 Calculus
.1 Infinitesimal calculus
.11 Definition of $\lim F(n)\ n \to \infty$
.12 Series; convergence
.13 Tests for convergence
.14 $\lim_{n \to a} F(n)$
.15 Continuity
.16 Uniform convergence
.17 Infinite integral. Convergence
.2 Differential calculus
.200 Differentiation and derivatives
.201 Partial differentiation
.21 Series
.210 Use of Taylor's and Maclaurin's theorems assuming their existence
.211 Binomial series, logarithms, exponential series
.212 Existence of Taylor's series and the remainder after n terms
.213 Series for sin x, cos x
.214 Arc tan x
.215 Arithmetic geometric series
.216 Summation of 'finite series'; indnction
.217 Binomial theorems for positive integral exponent
.27 Variation of function values
.271 Rolle's theorem for polynomial functions, with proof
.272 Rolle's theorem in general (intuitive approach but precise statement)
.273 Maxima, minima, inflexions
.274 Order of contact
.29 Differential calculus problems
.291 Curvature
.292 Evolutes, involutes, orthogonal trajectories, envelopes
.293 Leibnitz's theorem on differentiation of product
.3 Integral calculus
.30 Definition and properties of the definite integral
.31 Methods of integration
.312 Techniques of integration. Substitution, partial fractions, parts, reduction formulae
.313 Concept of integrating as the inverse of differentiating
.38 Differential equations

$$\frac{dy}{dx} = ky$$

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c = 0$$

$$\frac{d^2y}{dx^2} + n^2y = 0$$

$$\frac{dy}{dx} + Py = Q$$

$$P(y)\,dy = Q(x)\,dx$$

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

.39 Integral calculus problems and tables
.390 Areas assuming existence
.391 Volumes of solids of revolution
.392 Centroids, moments of inertia
.393 Investigation of existence of area
.394 Length of arc
.395 Areas of surface of revolution
.5 Calculus of functions
.52 Functions of r al variables
521 Logarithms and exponential functions
.522 Circular functions
.523 Hyperbolic functions
.524 Inverse circular functions
.525 Inverse hyperbolic functions
.6 Calculus of finite differences
.61 Numerical methods
.62 Solution of numerical equations, Newton, Morner etc.
.63 Evaluation of definite integrals. Simpson, Gauss etc.
.64 Approx to functions by polynomials
.8 Functions of complex variables
.81 Exponential, logarithmic functions of complex variables
519 Probabilities and Statistical Mathematics
.1 Probabilities
Fundamental laws of probability
.8 Treatment of data
Presentation; pie charts, histograms; frequency, polygons
.9 Statistical mathematics
.900 Mean, mode, median, deciles, percentiles
.901 Measures of dispersion, variance
.902 Binomial distribution
.903 Poisson distribution
904 Normal (Gaussian) distribution
.905 Sampling: estimation of population parameters
.906 Tests for significance for small samples
.907 t test, F test, difference of means etc.
.908 Bivariate normal distribution
.909 Regressive lines; method of least squares
.910 Product moment correlation
.911 Ranking methods
.912 Time series; moving averages, standardised rates (death rates etc.)
.913 S mple analyses of variance techniques
.92 Programming
Simple linear programmes
531.1 Kinematics
Motion in a circle with uniform speed
Motion of a particle (heavy) constrained to move in a vertical circle
Radial and transverse components of acceleration
Tangential and normal components
Motion in a plane with acceleration towards a fixed point
Orbits
531.32 *Simple harmonic motions*
Simple harmonic motion
Simple sine wave motion
Combinations of SHMs in a plane — equal frequencies

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